

# $D = 3 \mathcal{N} = 6$ superconformal symmetry of $AdS_4 \times \mathbb{CP}^3$ superstring

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## Abstract

Invariance of the  $AdS_4 \times \mathbb{CP}^3$  superstring under  $D = 3 \mathcal{N} = 6$  superconformal symmetry is discussed in the sector described by the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset sigma-model action presented in the conformal basis for the  $osp(4|6)/(so(1,3) \times u(3))$  Cartan forms. Transformation rules under  $D = 3 \mathcal{N} = 6$  superconformal symmetry for the  $(10|24)$ -dimensional 'reduced'  $AdS_4 \times \mathbb{CP}^3$  superspace coordinates are obtained and used to derive corresponding world-sheet currents.

## 1 Introduction

The first explicit example of the gauge/string duality [1], [2], [3] allowed to probe analytically previously inaccessible nonperturbative regime of  $\mathcal{N} = 4$  super-Yang-Mills theory via the *IIB* superstring on  $AdS_5 \times S^5$  background. It is in a sense the simplest instance of the *AdS/CFT* correspondence due to the maximal supersymmetry described by  $PSU(2,2|4)$  supergroup both of the  $AdS_5 \times S^5$  superbackground and  $D = 4$  supersymmetric conformal field theory (SCFT) on the boundary of  $AdS_5$  space.

Another highly supersymmetric explicit example of the *AdS/CFT* correspondence that was put forward not long ago by Aharony, Bergman, Jafferis and Maldacena (ABJM) [4] provides dual description of the SCFT in the space-time of one lower dimension in terms of *M*-theory on  $AdS_4 \times (S^7/\mathbb{Z}_k)$  background. In spite of the fact that lower dimensional theories basically have simpler dynamics compared to  $4d$  ones the ABJM correspondence appears to be harder to verify since on both sides of the duality the isometry supergroup  $OSp(4|6)$  isomorphic to  $D = 3 \mathcal{N} = 6$  superconformal symmetry is lower than the maximally allowed one.

Difficulties manifest itself already at the level of constructing the classical action for the *IIA* superstring on  $AdS_4 \times \mathbb{CP}^3$  superbackground that provides dual description of the 't Hooft limit of  $D = 3$  SCFT proposed by ABJM [4]. Group-theoretic supercoset approach [5], [6], [7], [8] originally elaborated to describe the *IIB* superstring on  $AdS_5 \times S^5$  background when applied to the  $AdS_4 \times \mathbb{CP}^3$  superstring gives the partial answer [9], [10]<sup>2</sup> because only the subspace of  $AdS_4 \times \mathbb{CP}^3$  superspace can be realized as the supercoset manifold  $OSp(4|6)/(SO(1,3) \times U(3))$ . The  $OSp(4|6)/(SO(1,3) \times U(3))$  sigma-model action [9], [10] corresponds to fixing half of the gauge freedom related to  $\kappa$ -symmetry of the complete action [12] that can be obtained via the double dimensional reduction [13] of the  $D = 11$  supermembrane action on the maximally supersymmetric  $AdS_4 \times S^7$  background [14] due to the Hopf fibration realization of the 7-sphere  $S^7 = \mathbb{CP}^3 \times S^1$  [15], [16]. Although  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset sigma-model fails to describe all possible  $AdS_4 \times \mathbb{CP}^3$  superstring configurations [9], [12] it has clear group-theoretical structure and is classically

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<sup>2</sup>Alternative way to construct the  $AdS_4 \times \mathbb{CP}^3$  superstring using the pure spinor approach was followed in [11].

integrable allowing one to utilize for its investigation many of the results obtained for the "elder brother" example of  $AdS_5/CFT_4$  correspondence relying on the integrable structures<sup>3</sup> exhibited there [18], [19] (for the collection of recent reviews see, e.g., [20]).

In Ref. [21] we have found explicit form of the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset sigma-model action in the conformal basis for  $osp(4|6)$  Cartan forms considering the supercoset representative parametrized by Poincare coordinates for the  $AdS_4$  space with 24 fermionic coordinates split into two sets of 12 related to Poincare and conformal supersymmetries from the  $AdS$  boundary superspace perspective<sup>4</sup>. Such a choice of the supercoset representative allows to formulate the stringy side of the duality in terms of the variables that contain those parametrizing  $D = 3$   $\mathcal{N} = 6$  boundary superspace, where the ABJM theory could be formulated off-shell [26], [27], [28] aiming at getting new insights into the relation between both theories.

The goal of this paper is to establish transformation properties under  $D = 3$   $\mathcal{N} = 6$  superconformal symmetry of the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset coordinates introduced in Ref. [21], as well as to find Noether currents associated with the  $D = 3$   $\mathcal{N} = 6$  superconformal invariance of the  $OSp(4|6)/(SO(1,3) \times U(3))$  superstring action. Similar problem of deriving  $D = 4$   $\mathcal{N} = 4$  superconformal transformations for the  $AdS_5 \times S^5$  superspace coordinates relevant to the  $AdS_5/CFT_4$  correspondence was addressed in [29], [30]. We start with reviewing the  $OSp(4|6)/(SO(1,3) \times U(3))$  sigma-model in the conformal basis for Cartan forms, then examine the action of left  $D = 3$   $\mathcal{N} = 6$  superconformal transformations on the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset element and proceed to derivation of the  $osp(4|6)$  Cartan forms transformation rules and Noether current densities for each of the individual transformations from  $D = 3$   $\mathcal{N} = 6$  superconformal symmetry.

## 2 $OSp(4|6)/(SO(1,3) \times U(3))$ superstring in conformal basis

The sigma-model action on the  $(10|24)$ -dimensional  $OSp(4|6)/(SO(1,3) \times U(3))$  superspace was found in [9], [10] following the general prescription [5], [6], [7], [8] for constructing sigma-model-type actions on supercoset spaces that admit 4-element outer automorphism  $\mathbb{Z}_4$  of the underlying isometry superalgebra. It relies on identifying Cartan forms associated with 10 bosonic and 24 fermionic generators of the  $osp(4|6)/(so(1,3) \times u(3))$  supercoset as the  $(10|24)$ -dimensional supervielbein components. Resulting action is invariant under the global  $OSp(4|6)$  supersymmetry, as well as gauge  $SO(1,3) \times U(3)$  and  $\kappa$ -symmetries, describes the requisite number of physical degrees of freedom, has correct bosonic limit and is classically integrable.

In Ref.[21] we have considered the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset element

$$\mathcal{G} = e^{x^m P_m + \theta_a^\mu Q_\mu^a + \bar{\theta}^{\mu a} \bar{Q}_{\mu a}} e^{\eta_{\mu a} S^{\mu a} + \bar{\eta}_\mu^a \bar{S}_a^\mu} e^{z^a V_a + \bar{z}_a \bar{V}_a} e^{\varphi D} \quad (2.1)$$

parametrized by  $D = 3$   $\mathcal{N} = 6$  super-Poincare coordinates  $(x^m, \theta_a^\mu, \bar{\theta}^{\mu a})$ ,  $AdS_4$  radial direction coordinate  $\varphi$  related to the boundary-space dilatations, 3 complex coordinates  $(z^a, \bar{z}_a)$  of the  $\mathbb{CP}^3$  manifold, and 12 fermionic coordinates  $(\eta_{\mu a}, \bar{\eta}_\mu^a)$  corresponding to  $D = 3$   $\mathcal{N} = 6$

<sup>3</sup>The issue of seeking possible integrable structures for the  $AdS_4 \times \mathbb{CP}^3$  superstring beyond the  $OSp(4|6)/(SO(1,3) \times U(3))$  sigma-model has been recently addressed in Ref.[17].

<sup>4</sup>Previously such conformal-type parametrizations were used to examine the string/brane models involved into the higher-dimensional examples of  $AdS/CFT$  correspondence [22], [23], [24], [25].

conformal supersymmetry. Associated current 1-form in the conformal basis reads

$$\begin{aligned}\mathcal{C}(d) = \mathcal{G}^{-1}d\mathcal{G} = & \hat{\omega}^m(d)P_m + \hat{c}^m(d)K_m + \Delta(d)D + G^{mn}(d)M_{mn} \\ & + \Omega_a{}^4(d)V_4^a + \Omega_4{}^a(d)V_a{}^4 + \Omega_a{}^b(d)V_b{}^a + \Omega_4{}^4(d)V_4{}^4 \\ & + \hat{\omega}_a^\mu(d)Q_\mu^a + \hat{\omega}^{\mu a}(d)\bar{Q}_{\mu a} + \hat{\chi}_{\mu a}(d)S^{\mu a} + \hat{\chi}_\mu^a(d)\bar{S}_a^\mu\end{aligned}\quad (2.2)$$

or manifesting the  $\mathbb{Z}_4$ -grading

$$\mathcal{C}(d) = \mathcal{C}_0(d) + \mathcal{C}_2(d) + \mathcal{C}_1(d) + \mathcal{C}_3(d), \quad (2.3)$$

where

$$\begin{aligned}\mathcal{C}_0(d) = & \frac{1}{2}(\hat{\omega}^m(d) - \hat{c}^m(d))(P_m - K_m) + G^{mn}(d)M_{mn} + \Omega_a{}^b(d)V_b{}^a + \Omega_4{}^4(d)V_4{}^4, \\ \mathcal{C}_2(d) = & \frac{1}{2}(\hat{\omega}^m(d) + \hat{c}^m(d))(P_m + K_m) + \Delta(d)D + \Omega_a{}^4(d)V_4^a + \Omega_4{}^a(d)V_a{}^4, \\ \mathcal{C}_1(d) = & \frac{1}{2}(\hat{\omega}_a^\mu(d) + i\hat{\chi}_a^\mu(d))(Q_\mu^a + iS_\mu^a) + \frac{1}{2}(\hat{\omega}^{\mu a}(d) - i\hat{\chi}^{\mu a}(d))(\bar{Q}_{\mu a} - i\bar{S}_{\mu a}), \\ \mathcal{C}_3(d) = & \frac{1}{2}(\hat{\omega}_a^\mu(d) - i\hat{\chi}_a^\mu(d))(Q_\mu^a - iS_\mu^a) + \frac{1}{2}(\hat{\omega}^{\mu a}(d) + i\hat{\chi}^{\mu a}(d))(\bar{Q}_{\mu a} + i\bar{S}_{\mu a}).\end{aligned}\quad (2.4)$$

Then the  $\mathbb{Z}_4$ -invariant  $OSp(4|6)/(SO(1,3) \times U(3))$  superstring action in the conformal basis for Cartan forms (2.2) acquires the form

$$S = -\frac{1}{2} \int d^2\xi \sqrt{-g} g^{ij} \left[ \frac{1}{4}(\hat{\omega}_i^m + \hat{c}_i^m)(\hat{\omega}_{jm} + \hat{c}_{jm}) + \Delta_i \Delta_j + \frac{1}{2}(\Omega_{ia}{}^4 \Omega_{j4}{}^a + \Omega_{ja}{}^4 \Omega_{i4}{}^a) \right] + S_{WZ} \quad (2.5)$$

with the Wess-Zumino action given by

$$\begin{aligned}S_{WZ} = & -\frac{1}{4}\varepsilon^{ij} \int d^2\xi \left[ (\hat{\omega}_{ia}^\mu + i\hat{\chi}_{ia}^\mu)\varepsilon_{\mu\nu}(\hat{\omega}_j^{\nu a} + i\hat{\chi}_j^{\nu a}) + (\hat{\omega}_{ia}^\mu - i\hat{\chi}_{ia}^\mu)\varepsilon_{\mu\nu}(\hat{\omega}_j^{\nu a} - i\hat{\chi}_j^{\nu a}) \right] \\ = & -\frac{1}{2}\varepsilon^{ij} \int d^2\xi \left( \hat{\omega}_{ia}^\mu \varepsilon_{\mu\nu} \hat{\omega}_j^{\nu a} + \hat{\chi}_{i\mu a} \varepsilon^{\mu\nu} \hat{\chi}_{j\nu}^a \right).\end{aligned}\quad (2.6)$$

The first two summands in the kinetic part of the action (2.5) include Cartan forms associated with the generators  $P_m$  of the space-time translations on the  $D = 3$  Minkowski boundary of  $AdS_4$  space

$$\hat{\omega}^m(d) = e^{-2\varphi}\omega^m(d), \quad \omega^m(d) = dx^m - id\theta_a^\mu \sigma_{\mu\nu}^m \bar{\theta}^{\nu a} + i\theta_a^\mu \sigma_{\mu\nu}^m d\bar{\theta}^{\nu a}, \quad (2.7)$$

conformal boost generators  $K_m$

$$\begin{aligned}\hat{c}^m(d) = & e^{2\varphi}c^m(d), \quad c^m(d) = -id\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}\bar{\eta}_\nu^a + i\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}d\bar{\eta}_\nu^a \\ & + 2(\bar{\eta}\eta) \left[ \eta_{\mu a}\tilde{\sigma}^{m\mu\nu}(d\bar{\theta}_\nu^a + \frac{1}{4}\bar{\zeta}_\nu^a(d)) - (d\theta_{\mu a} + \frac{1}{4}\zeta_{\mu a}(d))\tilde{\sigma}^{m\mu\nu}\bar{\eta}_\nu^a \right], \quad \bar{\eta}\eta \equiv \bar{\eta}_\rho^b \eta_b^\rho,\end{aligned}\quad (2.8)$$

where

$$\zeta_a^\mu(d) = -\tilde{\sigma}^{m\mu\nu}\omega_m(d)\eta_{\nu a} = -\tilde{\omega}^{\mu\nu}(d)\eta_{\nu a}, \quad \bar{\zeta}^{\mu a}(d) = -\tilde{\sigma}^{m\mu\nu}\omega_m(d)\bar{\eta}_\nu^a = -\tilde{\omega}^{\mu\nu}(d)\bar{\eta}_\nu^a, \quad (2.9)$$

and dilatations

$$\Delta(d) = d\varphi + i(d\theta_a^\mu \bar{\eta}_\mu^a + d\bar{\theta}^{\mu a} \eta_{\mu a}) \quad (2.10)$$

with the corresponding generator  $D$ . Note that the generators  $(D, P_m + K_m)$  can be identified as the  $so(2,3)/so(1,3)$  coset generators<sup>5</sup> and corresponding Cartan forms represent the  $AdS$  part of the  $(10|24)$ -supervielbein bosonic components.

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<sup>5</sup>(Anti)commutation relations of the  $D = 3$   $\mathcal{N} = 6$  superconformal algebra can be found in Ref.[21].

Supervielbein components in the directions tangent to the  $\mathbb{CP}^3$  manifold are identified with the  $su(4)/u(3)$  Cartan forms  $(\Omega_a^4(d), \Omega_4^a(d))$  that are the off-diagonal components of the traceless Hermitean matrix of  $su(4)$  Cartan forms

$$\Omega_A^B(d) = \begin{pmatrix} \Omega_a^b & \Omega_a^4 \\ \Omega_4^b & \Omega_4^4 \end{pmatrix}, \quad \Omega_4^4 = -\Omega_a^a. \quad (2.11)$$

Using the isomorphism  $SU(4) \sim SO(6)$  it is possible to accommodate  $su(4)$  Cartan forms into the  $6 \times 6$  matrix

$$\Omega_{\hat{a}}^{\hat{b}}(d) = \begin{pmatrix} \Omega_a^b - \delta_a^b \Omega_c^c & \varepsilon_{acb} \Omega_4^c \\ -\varepsilon^{acb} \Omega_c^4 & -\Omega_b^a + \delta_b^a \Omega_c^c \end{pmatrix} \quad (2.12)$$

antisymmetric w.r.t the metric

$$H_{\hat{a}\hat{b}} = \begin{pmatrix} 0 & \delta_a^b \\ \delta_b^a & 0 \end{pmatrix} \quad (2.13)$$

following the decomposition of the  $D = 6$  vector representation as  $\mathbf{3} \oplus \bar{\mathbf{3}}$  of  $SU(3)$ <sup>6</sup>. For the considered choice of the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset element  $su(4)$  Cartan forms are given by the sum of two contributions

$$\Omega_{\hat{a}}^{\hat{b}}(d) = \Omega_{\mathbf{b}\hat{a}}^{\hat{b}}(d) + \Omega_{\mathbf{f}\hat{a}}^{\hat{b}}(d) \quad (2.14)$$

coming from bosons and fermions. Bosonic contribution

$$\Omega_{\mathbf{b}\hat{a}}^{\hat{b}}(d) = iT_{\hat{a}}^{\hat{c}} d\bar{T}_{\hat{c}}^{\hat{b}} \quad (2.15)$$

is described by the  $su(4)$  Cartan form matrix associated with the  $SU(4)/U(3)$  coset element

$$T_{\hat{a}}^{\hat{b}} = \begin{pmatrix} T_a^b & T_{ab} \\ T^{ab} & T_b^a \end{pmatrix} = \exp \begin{pmatrix} 0 & i\varepsilon_{acb} z^c \\ -i\varepsilon^{acb} \bar{z}_c & 0 \end{pmatrix}. \quad (2.16)$$

Explicit expressions for the purely bosonic part of  $su(4)$  Cartan forms can be found in [21]. Fermionic contribution to (2.14)

$$\Omega_{\mathbf{f}\hat{a}}^{\hat{b}}(d) = (T\Psi(d)\bar{T})_{\hat{a}}^{\hat{b}} \quad (2.17)$$

is obtained by the  $T$ -transformation of the matrix

$$\Psi_{\hat{a}}^{\hat{b}}(d) = 2(d\theta_{\hat{a}}^{\mu} \eta_{\mu}^{\hat{b}} - d\theta^{\mu\hat{b}} \eta_{\mu\hat{a}} - \eta_{\mu\hat{a}} \tilde{\omega}^{\mu\nu}(d) \eta_{\nu}^{\hat{b}}), \quad (2.18)$$

where the Grassmann coordinates have been written as  $D = 6$  vectors in the  $\mathbf{3} \oplus \bar{\mathbf{3}}$  basis

$$\theta_{\hat{a}}^{\mu} = \begin{pmatrix} \theta_a^{\mu} \\ \bar{\theta}_{\mu a} \end{pmatrix}, \quad \eta_{\mu\hat{a}} = \begin{pmatrix} \eta_{\mu a} \\ \bar{\eta}_{\mu}^a \end{pmatrix} \quad (2.19)$$

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<sup>6</sup>The metric  $H_{\hat{a}\hat{b}}$  is the conventional unit  $D = 6$  metric  $\delta^{IJ}$  written in the  $\mathbf{3} \oplus \bar{\mathbf{3}}$  basis. Both bases are connected by the transformation matrices

$$M^{I\hat{a}} = \frac{1}{2}(\tilde{\rho}^{Ia4}, \rho_{a4}^I), \quad M^{-1}_{\hat{a}I} = \begin{pmatrix} \rho_{a4}^I \\ \tilde{\rho}^{I4a} \end{pmatrix}: \quad MM^{-1} = I,$$

where  $\rho_{AB}^I$  and  $\tilde{\rho}^{IAB}$  are  $D = 6$  chiral  $\gamma$ -matrices, such that the components of a  $D = 6$  vector  $O^I$  in these bases can be transformed into one another  $O^I = M^{I\hat{a}} O_{\hat{a}}$  and  $O_{\hat{a}} = M^{-1}_{\hat{a}I} O^I$ . In particular, for the  $D = 6$  metric we find that  $\delta^{IJ} = -2M^{I\hat{a}} H_{\hat{a}\hat{b}} M^{J\hat{b}}$ .

and  $\theta^{\mu\hat{a}} = H^{\hat{a}\hat{b}}\theta_{\hat{b}}^{\mu}$ ,  $\eta_{\mu}^{\hat{a}} = H^{\hat{a}\hat{b}}\eta_{\mu\hat{b}}$ . The  $T$ -transformed  $\mathbf{3} \oplus \bar{\mathbf{3}}$  vectors will be endowed with hats  $\hat{\theta}_{\hat{a}}^{\mu} = T_{\hat{a}}^{\hat{b}}\theta_{\hat{b}}^{\mu}$ ,  $\hat{\theta}^{\mu\hat{a}} = H^{\hat{a}\hat{b}}\hat{\theta}_{\hat{b}}^{\mu}$  etc. Using that  $H_{\hat{a}\hat{c}}(\bar{T}^T)^{\hat{c}}_{\hat{d}}H^{\hat{d}\hat{b}} = T_{\hat{a}}^{\hat{b}}$  for the chosen realization of the matrix  $T$  the fermionic part of  $su(4)$  Cartan form matrix can be cast into the form

$$\Omega_{\mathbf{f}\hat{a}}^{\hat{b}}(d) = 2(\hat{d}\theta_{\hat{a}}^{\mu}\hat{\eta}_{\mu}^{\hat{b}} - \hat{d}\theta^{\mu\hat{b}}\hat{\eta}_{\mu\hat{a}} - \hat{\eta}_{\mu\hat{a}}\hat{\omega}^{\mu\nu}(d)\hat{\eta}_{\nu}^{\hat{b}}). \quad (2.20)$$

The WZ term of the action (2.5) in the  $\mathbf{3} \oplus \bar{\mathbf{3}}$  basis can be written in the form

$$S_{WZ} = -\frac{i}{8}\varepsilon^{ij}\mathfrak{J}_{\hat{a}}^{\hat{b}} \int d^2\xi \left( \hat{\omega}_i^{\mu\hat{a}}\varepsilon_{\mu\nu}\hat{\omega}_{j\hat{b}}^{\nu} + \hat{\chi}_{i\mu}^{\hat{a}}\varepsilon^{\mu\nu}\hat{\chi}_{j\nu\hat{b}} \right), \quad (2.21)$$

where  $\mathfrak{J}_{\hat{a}}^{\hat{b}}$  is the  $\mathbb{CP}^3$  Kahler 2-form written in the  $\mathbf{3} \oplus \bar{\mathbf{3}}$  basis<sup>7</sup>

$$\mathfrak{J}_{\hat{a}}^{\hat{b}} = 2i \begin{pmatrix} \delta_{\hat{a}}^{\hat{b}} & 0 \\ 0 & -\delta_{\hat{b}}^{\hat{a}} \end{pmatrix}. \quad (2.22)$$

It contains the world-sheet projections of fermionic 1-forms

$$\hat{\omega}_{\hat{a}}^{\mu}(d) = e^{-\varphi}T_{\hat{a}}^{\hat{b}}\omega_{\hat{b}}^{\mu}(d), \quad \omega_{\hat{b}}^{\mu}(d) = d\theta_{\hat{b}}^{\mu} + \zeta_{\hat{b}}^{\mu}(d) \quad (2.23)$$

related to Poincare supersymmetry generators  $(Q_{\mu}^a, \bar{Q}_{\mu a})$ , and

$$\hat{\chi}_{\mu\hat{a}}(d) = e^{\varphi}T_{\hat{a}}^{\hat{b}}\chi_{\mu\hat{b}}(d), \quad \chi_{\mu\hat{a}}(d) = d\eta_{\mu\hat{a}} + 2i\eta_{\mu}^{\hat{b}}d\theta_{\hat{b}}^{\nu}\eta_{\nu\hat{a}} - i(\bar{\eta}\eta)\omega_{\mu\hat{a}}(d) \quad (2.24)$$

related to conformal supersymmetry generators  $(S^{\mu a}, \bar{S}_a^{\mu})$ .

### 3 $D = 3$ $\mathcal{N} = 6$ superconformal symmetry of the $OSp(4|6)/(SO(1,3) \times U(3))$ superstring: general properties and coordinate transformations

Global  $OSp(4|6)$  transformations act on the  $OSp(4|6)/(SO(1,3) \times U(3))$  coset representative from which the left-invariant Cartan forms (2.2) are constructed in the following way<sup>8</sup>

$$\mathcal{G}'H = G\mathcal{G}, \quad G \in OSp(4|6) \quad (3.1)$$

with  $H$  being the compensating  $SO(1,3) \times U(3)$  transformation or passing to infinitesimal parameters

$$\delta\mathcal{G} = g\mathcal{G} - \mathcal{G}h, \quad g \in osp(4|6), \quad h \in so(1,3) \oplus u(3). \quad (3.2)$$

Substituting above relation into (2.2) yields

$$\mathcal{C}(\delta) = \mathcal{G}^{-1}\delta\mathcal{G} = \mathcal{G}^{-1}g\mathcal{G} - h. \quad (3.3)$$

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<sup>7</sup>In conventional  $D = 6$  vector basis it is given by the expression  $J^{IJ} = \frac{i}{2}(\rho_{4a}^I\tilde{\rho}^{J4a} - \rho_{4a}^J\tilde{\rho}^{I4a})$ . It takes simple diagonal form when contracted with the  $6d$  rotation generators

$$J_A{}^B = J^{IJ}\rho^I{}_A{}^J{}_B = \begin{pmatrix} -2i & 0 \\ 0 & 6i \end{pmatrix}.$$

The matrix  $J_A{}^B$  can be shown to satisfy the following equation  $J_A{}^C J_C{}^B - 4iJ_A{}^B + 12\delta_A^B = 0$ .

<sup>8</sup>Since the supercoset string action is built out of the Cartan forms it is exactly invariant under the global symmetry in distinction with the original Green-Schwarz action [31] that is quasi-invariant because its WZ term cannot be presented as a 2-form in supercurrents. For detailed discussion on that point and the properties of WZ term on  $AdS$  backgrounds see, e.g., [32].

Consider the  $D = 3$   $\mathcal{N} = 6$  superconformal algebra valued transformation parameter

$$g = a^m P_m + b_m K^m + f D + \frac{1}{2} l^{mn} M_{mn} + y^a V_a^4 + \bar{y}_a V_4^a + w_a^b V_b^a + w_4^a V_4^a + \varepsilon_a^\mu Q_\mu^a + \bar{\varepsilon}^{\mu a} \bar{Q}_{\mu a} + \xi_{\mu a} S^{\mu a} + \bar{\xi}_\mu^a \bar{S}_a^\mu. \quad (3.4)$$

It includes the parameters of  $D = 3$  Minkowski space-time translations  $a^m$ , conformal boosts  $b_m$ , dilatations  $f$  and Lorentz rotations  $l^{mn}$ , as well as anticommuting parameters of  $D = 3$   $\mathcal{N} = 6$  Poincare supersymmetry  $(\varepsilon_a^\mu, \bar{\varepsilon}^{\mu a})$  and conformal supersymmetry  $(\xi_{\mu a}, \bar{\xi}_\mu^a)$  supplemented by the  $SU(4)$   $R$ -symmetry parameters  $(w_a^b, y^a, \bar{y}_a)$ . Then the substitution of  $OSp(4|6)/(SO(1,3) \times U(3))$  coset representative (2.1) into (3.3) yields

$$\begin{aligned} \mathcal{C}(\delta) = & (\hat{\omega}^m(\delta) - \hat{b}^m) P_m + (\hat{c}^m(\delta) + \hat{b}^m) K_m + \Delta(\delta) D + (G^{mn}(\delta) + \frac{1}{2} \hat{l}^{mn}) M_{mn} \\ & + \Omega_4^a(\delta) V_a^4 + \Omega_a^4(\delta) V_4^a + (\Omega_a^b(\delta) + \hat{w}_a^b) V_b^a + (\Omega_4^a(\delta) + \hat{w}_4^a) V_4^a \\ & + \hat{\omega}_a^\mu(\delta) Q_\mu^a + \hat{\omega}^{\mu a}(\delta) Q_{\mu a} + \hat{\chi}_{\mu a}(\delta) S^{\mu a} + \hat{\chi}_\mu^a(\delta) \bar{S}_a^\mu. \end{aligned} \quad (3.5)$$

The quantities that cannot be accommodated into the individual Cartan form variations like, e.g.  $\hat{\omega}^m(\delta) = i_\delta \hat{\omega}^m(d)$  represent parameters of the compensating transformations. In particular, the vector

$$\hat{b}^m = e^{2\varphi} A^{-1} b^m(\theta), \quad A = 1 - e^{4\varphi} (\bar{\eta} \eta)^2, \quad (3.6)$$

where

$$b^m(\theta) = b^m - i [(\xi_a(\theta) \tilde{\sigma}^m \bar{\eta}^a) + (\bar{\xi}^a(\theta) \tilde{\sigma}^m \eta_a)], \quad \xi_{\mu a}(\theta) = \xi_{\mu a} + b_{\mu\nu} \theta_a^\nu, \quad (3.7)$$

is the parameter of  $SO(1,3)/SO(1,2)$  transformations, while the antisymmetric tensor

$$\hat{l}^{mn} = l^{mn}(\theta) + i e^{2\varphi} [(\eta_a \hat{b} \sigma^{mn} \bar{\eta}^a) + (\bar{\eta}^a \hat{b} \sigma^{mn} \eta_a)] \quad (3.8)$$

with

$$l^{mn}(\theta) = l^{mn} + 2(b^m x^n - b^n x^m) + 2i[(\theta_a \sigma^{mn} \bar{\xi}^a) + (\bar{\theta}^a \sigma^{mn} \xi_a)] + i[(\theta_a \sigma^{mn} b \bar{\theta}^a) + (\bar{\theta}^a \sigma^{mn} b \theta_a)] \quad (3.9)$$

describes compensating  $SO(1,2)$  Lorentz rotations. The parameters of compensating  $U(3)$  rotations

$$\hat{w}_a^b = \tilde{w}_a^b + \frac{i(1-\cos|z|)}{|z|\sin|z|} (\tilde{z}_a \tilde{y}^b - \tilde{y}_a z^b) + \frac{i(1-\cos|z|)^2}{2|z|^3 \cos|z|\sin|z|} ((\tilde{y} \tilde{z}) - (z \tilde{y})) \tilde{z}_a z^b, \quad |z|^2 = z^a \bar{z}_a \quad (3.10)$$

have been presented in the form exhibiting explicit dependence on the entries

$$\begin{aligned} \tilde{w}_a^b &= w_a^b(\theta) - e^{2\varphi} [2(\eta_a \hat{b} \bar{\eta}^b) - \delta_a^b (\eta_c \hat{b} \bar{\eta}^c)], \\ \tilde{y}^a &= y^a(\theta) + e^{2\varphi} \varepsilon^{abc} (\eta_b \hat{b} \bar{\eta}_c), \quad \tilde{y}_a = \bar{y}_a(\theta) - e^{2\varphi} \varepsilon_{abc} (\bar{\eta}^b \hat{b} \bar{\eta}^c), \end{aligned} \quad (3.11)$$

where

$$\begin{aligned} w_a^b(\theta) &= w_a^b - 2(\xi_{\mu a} \bar{\theta}^{\mu b} + \theta_a^\mu \bar{\xi}_\mu^b) + \delta_a^b (\xi_{\mu c} \bar{\theta}^{\mu c} + \theta_c^\mu \bar{\xi}_\mu^c) - 2(\theta_a b \bar{\theta}^b) + \delta_a^b (\theta_c b \bar{\theta}^c), \\ y^a(\theta) &= y^a + \varepsilon^{abc} (2\xi_{\mu b} \theta_c^\mu + (\theta_b b \bar{\theta}_c)), \quad \bar{y}_a(\theta) = \bar{y}_a - \varepsilon_{abc} (2\bar{\xi}_\mu^b \bar{\theta}^{\mu c} + (\bar{\theta}^b b \bar{\theta}^c)), \end{aligned} \quad (3.12)$$

of the  $SU(4)$  matrix

$$\widetilde{W}_a^{\hat{b}} = \begin{pmatrix} \tilde{w}_a^b - \delta_a^b \tilde{w}_c^c & \varepsilon_{acb} \tilde{y}^c \\ -\varepsilon^{acb} \tilde{y}_c & -\tilde{w}_b^a + \delta_b^a \tilde{w}_c^c \end{pmatrix} \quad (3.13)$$

that, as will be shown below, enters the transformation laws (3.18) of the  $SU(4)/U(3)$  coset element (2.16) under the  $D = 3$   $\mathcal{N} = 6$  superconformal symmetry.

$D = 3 \mathcal{N} = 6$  superconformal transformations of the  $OSp(4|6)/(SO(1,3) \times U(3))$  superspace coordinates that parametrize (2.1) include also the contributions proportional to the parameters of the compensating transformations (3.6), (3.8) and (3.10).  $D = 3 \mathcal{N} = 6$  Boundary superspace coordinates obey the following transformation rules<sup>9</sup>

$$\begin{aligned} \delta x^m = & a^m + l^m{}_n x^n + 2f x^m + b^m(x^2 + (\bar{\theta}\theta)^2) - 2x^m b_n x^n \\ & - i[(\varepsilon_a \sigma^m \bar{\theta}^a) + (\bar{\varepsilon}^a \sigma^m \theta_a)] - i[(\xi_a \hat{x} \sigma^m \bar{\theta}^a) + (\bar{\xi}^a \hat{x} \sigma^m \theta_a)] \\ & + e^{2\varphi} \left\{ \hat{b}^m + i[(\eta_a \hat{b} \sigma^m \bar{\theta}^a) + (\bar{\eta}^a \hat{b} \sigma^m \theta_a)] \right\}, \end{aligned} \quad (3.14)$$

$$\begin{aligned} \delta \theta_a^\mu = & \varepsilon_a^\mu + \frac{1}{4} l^{mn} \theta_a^\nu \sigma_{mn\nu}{}^\mu + f \theta_a^\mu + i w_b{}^b \theta_a^\mu - i w_a{}^b \theta_b^\mu - i \varepsilon_{abc} y^b \bar{\theta}^{\mu c} + \hat{x}^{\mu\nu} b_{\nu\lambda} \theta_a^\lambda \\ & + \hat{x}^{\mu\nu} \xi_{\nu a} - 2i(\theta_b^\mu \bar{\xi}_\nu^b + \bar{\theta}^{\mu b} \xi_{\nu b}) \theta_a^\nu + e^{2\varphi} \hat{b}^{\mu\nu} \eta_{\nu a} \end{aligned} \quad (3.15)$$

and c.c., where  $\hat{x}^{\mu\nu} = \tilde{x}^{\mu\nu} - i\varepsilon^{\mu\nu}(\bar{\theta}\theta)$ , while that for the coordinate  $\varphi$  related to the  $AdS_4$  space bulk direction reads

$$\delta \varphi = f(\theta) = f - b_m x^m + i(\xi_{\mu a} \bar{\theta}^{\mu a} + \bar{\xi}_\mu^a \theta_a^\mu). \quad (3.16)$$

Transformation properties of the  $\mathbb{CP}^3$  complex coordinates

$$\begin{aligned} \delta z^a = & i z^b \tilde{w}_b{}^a + i \tilde{w}_b{}^b z^a + \frac{|z| \cos |z|}{\sin |z|} \tilde{y}^a + \frac{1}{2|z|^2} \left( 1 - \frac{|z|}{\cos |z| \sin |z|} \right) (\tilde{y} \tilde{z}) z^a \\ & + \frac{1}{2|z|^2} [1 + |z|(\tan |z| - \cot |z|)] (z \tilde{y}) z^a \end{aligned} \quad (3.17)$$

can be summarized in the form of  $SU(4)/U(3)$  coset representative (2.16) transformations

$$\delta T_{\hat{a}}{}^{\hat{b}} = i(T_{\hat{a}}{}^{\hat{c}} \widetilde{W}_{\hat{c}}{}^{\hat{b}} - \widetilde{W}_{\hat{a}}{}^{\hat{c}} T_{\hat{c}}{}^{\hat{b}}), \quad \delta \bar{T}_{\hat{a}}{}^{\hat{b}} = -i(\widetilde{W}_{\hat{a}}{}^{\hat{c}} \bar{T}_{\hat{c}}{}^{\hat{b}} - \bar{T}_{\hat{a}}{}^{\hat{c}} \widetilde{W}_{\hat{c}}{}^{\hat{b}}), \quad (3.18)$$

where the  $SU(4)$  matrix  $\widetilde{W}_{\hat{a}}{}^{\hat{b}}$  has been introduced in (3.13) and

$$\widetilde{W}_{\hat{a}}{}^{\hat{b}} = \begin{pmatrix} \hat{w}_a{}^b - \delta_a^b \hat{w}_c{}^c & 0 \\ 0 & -\hat{w}_b{}^a + \delta_b^a \hat{w}_c{}^c \end{pmatrix} \quad (3.19)$$

represents  $U(3)$  compensating rotation matrix. Finally Grassmann coordinates associated with the conformal supersymmetry generators transform as follows

$$\begin{aligned} \delta \eta_{\mu a} = & \xi_{\mu a}(\theta) - \frac{1}{4} l^{mn}(\theta) \sigma_{mn\mu}{}^\nu \eta_{\nu a} - f(\theta) \eta_{\mu a} + i w_b{}^b(\theta) \eta_{\mu a} - i w_a{}^b(\theta) \eta_{\mu b} - i \varepsilon_{abc} y^b(\theta) \bar{\eta}_\mu^c \\ & + 2i e^{2\varphi} (\bar{\eta} \eta) \varepsilon_{\mu\lambda} \hat{b}^{\lambda\nu} \eta_{\nu a} \\ = & -\frac{1}{4} l^{mn} \sigma_{mn\mu}{}^\nu \eta_{\nu a} - f \eta_{\mu a} + i w_b{}^b \eta_{\mu a} - i w_a{}^b \eta_{\mu b} - i \varepsilon_{abc} y^b \bar{\eta}_\mu^c + b_{\mu\nu} \theta_a^\nu - \eta_{\nu a} \hat{x}^{\nu\lambda} b_{\lambda\mu} \\ & + 2i[(\theta_a b \bar{\theta}^b) \bar{\eta}_\mu^b + (\theta_a b \bar{\theta}^b) \eta_{\mu b}] + \xi_{\mu a} - 2i(\bar{\xi}_\mu^b \theta_b^\nu + \xi_{\mu b} \bar{\theta}^{\nu b}) \eta_{\nu a} \\ & - 2i(\eta_{\mu b} \bar{\xi}_\nu^b + \bar{\eta}_\mu^b \xi_{\nu b}) \theta_a^\nu + 2i \xi_{\nu a} (\theta_b^\nu \bar{\eta}_\mu^b + \bar{\theta}^{\nu b} \eta_{\mu b}) + 2i e^{2\varphi} (\bar{\eta} \eta) \varepsilon_{\mu\lambda} \hat{b}^{\lambda\nu} \eta_{\nu a} \end{aligned} \quad (3.20)$$

and c.c.

Cartan forms associated with the  $osp(4|6)/(so(1,3) \times u(3))$  supercoset generators are left-invariant under the above derived global transformations up to the compensating ones associated with the stability group generators. Corresponding Cartan forms in their turn

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<sup>9</sup>Observe vanishing of the terms proportional to  $SO(1,3)/SO(1,2)$  rotation parameters  $\hat{b}$  in the boundary limit  $\varphi \rightarrow -\infty$ .

transform in a connection-type way. In particular, bosonic 1-forms that are identified with the  $AdS_4$  part of the supervielbein in general transform as

$$\delta\hat{\omega}^m(d) + \delta\hat{c}^m(d) = \hat{l}^{mn}(\hat{\omega}_n(d) + \hat{c}_n(d)) + 4\hat{b}^m\Delta(d), \quad \delta\Delta(d) = -\hat{b}_m(\hat{\omega}^m(d) + \hat{c}^m(d)), \quad (3.21)$$

while, e.g.  $so(1,2)$  Cartan forms in the spinor realization  $G^{\mu\nu}(d) = \varepsilon^{\mu\lambda}\sigma_{mn\lambda}{}^\nu G^{mn}(d)$  obey the following rule

$$\delta G^{\mu\nu}(d) = \frac{1}{4}(G^{\mu\lambda}(d)\hat{l}_\lambda{}^\nu + G^{\nu\lambda}(d)\hat{l}_\lambda{}^\mu) + \hat{b}^\mu{}_\lambda(\hat{\omega}(d) - \hat{c}(d))^{\lambda\nu} + \hat{b}^\nu{}_\lambda(\hat{\omega}(d) - \hat{c}(d))^{\lambda\mu} - \frac{1}{2}d\hat{l}^{\mu\nu}. \quad (3.22)$$

$su(4)$  Cartan forms are  $OSp(4|6)$  left-invariant up to the  $U(3)$  gauge transformation

$$\delta\Omega_a{}^{\hat{b}}(d) = i(\Omega_a{}^{\hat{c}}(d)\widehat{W}_{\hat{c}}{}^{\hat{b}} - \widehat{W}_{\hat{a}}{}^{\hat{c}}\Omega_c{}^{\hat{b}}(d)) - d\widehat{W}_{\hat{a}}{}^{\hat{b}}, \quad (3.23)$$

from where we infer that the  $su(4)/u(3)$  1-forms identified with the  $\mathbb{CP}^3$  part of the supervielbein transform as

$$\delta\Omega_a{}^4(d) = i(\hat{w}_b{}^b\Omega_a{}^4(d) - \hat{w}_a{}^b\Omega_b{}^4(d)), \quad \delta\Omega_4{}^a(d) = -i(\hat{w}_b{}^b\Omega_4{}^a(d) - \Omega_4{}^b(d)\hat{w}_b{}^a) \quad (3.24)$$

and  $u(3)$  1-forms exhibit connection-type transformation properties

$$\delta\Omega_a{}^b(d) = i(\Omega_a{}^c(d)\hat{w}_c{}^b - \hat{w}_a{}^c\Omega_c{}^b(d)) - d\hat{w}_a{}^b. \quad (3.25)$$

Cartan forms that are identified with the supervielbein fermionic components transform in the following way

$$\delta\hat{\omega}_a{}^\nu(d) = \frac{1}{4}\hat{\omega}_a{}^\lambda(d)\hat{l}_\lambda{}^\nu + \hat{b}^{\nu\lambda}\hat{\chi}_{\lambda\hat{a}}(d) - i\widehat{W}_{\hat{a}}{}^{\hat{b}}\hat{\omega}_b{}^\nu(d) \quad (3.26)$$

and

$$\delta\hat{\chi}_{\nu\hat{a}}(d) = -\frac{1}{4}\hat{l}_\nu{}^\lambda\hat{\chi}_{\lambda\hat{a}}(d) + \hat{b}_{\nu\lambda}\hat{\omega}_a{}^\lambda(d) - i\widehat{W}_{\hat{a}}{}^{\hat{b}}\hat{\chi}_{\nu\hat{b}}(d). \quad (3.27)$$

For individual transformations from the  $D = 3$   $\mathcal{N} = 6$  superconformal symmetry to be discussed below these expressions simplify by properly restricting the parameters of compensating transformations that will be indicated by the vertical line.

## 4 $D = 3$ $\mathcal{N} = 6$ superconformal symmetry of the $OSp(4|6)/(SO(1,3) \times U(3))$ superstring: Noether currents

Noether current densities corresponding to the  $D = 3$   $\mathcal{N} = 6$  superconformal invariance of the superstring action (2.5) can be formally presented as the sum

$$\mathcal{J}_\Sigma^i(\tau, \sigma) = \mathcal{J}_{AdS_\Sigma}^i + \mathcal{J}_{CP_\Sigma}^i + \mathcal{J}_{WZ_\Sigma}^i, \quad (4.1)$$

where  $\Sigma$  is a transformation parameter<sup>10</sup>, of contributions of the  $AdS_4$

$$\mathcal{J}_{AdS_\Sigma}^i = -\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm} + \hat{c}_{jm})\frac{\partial}{\partial\Sigma}(\hat{\omega}^m(\delta_\Sigma) + \hat{c}^m(\delta_\Sigma)) + \Delta_j\frac{\partial}{\partial\Sigma}\Delta(\delta_\Sigma)\right) \quad (4.2)$$

and  $\mathbb{CP}^3$  parts of the kinetic term

$$\mathcal{J}_{CP_\Sigma}^i = -\frac{1}{2}\sqrt{-g}g^{ij}\left(\Omega_{ja}{}^4\frac{\partial}{\partial\Sigma}\Omega_4{}^a(\delta_\Sigma) + \Omega_{j4}{}^a\frac{\partial}{\partial\Sigma}\Omega_a{}^4(\delta_\Sigma)\right), \quad (4.3)$$

as well as that of the Wess-Zumino term

$$\mathcal{J}_{WZ_\Sigma}^i = \frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{a}}{}^{\hat{b}}\left(\hat{\omega}_j{}^{\mu\hat{a}}\varepsilon_{\mu\nu}\frac{\partial}{\partial\Sigma}\hat{\omega}_b{}^\nu(\delta_\Sigma) + \hat{\chi}_{j\mu}{}^{\hat{a}}\varepsilon^{\mu\nu}\frac{\partial}{\partial\Sigma}\hat{\chi}_{\nu\hat{b}}(\delta_\Sigma)\right). \quad (4.4)$$

Below we specialize to discussion of the individual transformations from the  $D = 3$   $\mathcal{N} = 6$  superconformal symmetry and present corresponding expressions for the Noether currents.

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<sup>10</sup>We assume the right derivative for fermions.



## 4.1 Noether currents associated with $D = 3$ conformal symmetry

### 4.1.1 Space-time translations

$osp(4|6)$  Cartan forms are obviously invariant under the global translations of  $D = 3$  Minkowski boundary coordinates. Their contributions to the current density are hence related to the coordinate dependence of transformation parameter. In particular, Eq.(3.21) representing variation of the Cartan forms identified with the  $AdS$  part of the supervielbein, when restricted to the boundary space-time translations acquires the form

$$\delta_a \hat{\omega}^m(d) + \delta_a \hat{c}^m(d) = j^m{}_n da^n, \quad \delta_a \Delta(d) = 0, \quad (4.5)$$

where the current contribution tensor equals

$$j^m{}_n = \frac{\partial(\hat{\omega}^m(\delta_a) + \hat{c}^m(\delta_a))}{\partial a^n} = e^{-2\varphi} A \delta_n^m. \quad (4.6)$$

$su(4)$  Cartan forms are also invariant under  $3d$  translations

$$\delta_a \Omega_{\hat{a}}{}^{\hat{b}}(d) = J_{\hat{a}}{}^{\hat{b}}{}_m da^m \quad (4.7)$$

modulo the current contribution matrix

$$J_{\hat{a}}{}^{\hat{b}}{}_m = \frac{\partial}{\partial a^m} \Omega_{\hat{a}}{}^{\hat{b}}(\delta_a) = \begin{pmatrix} j_a{}^{\hat{b}}{}_m & j_{abm} \\ -\bar{j}^{ab}{}_m & -j_b{}^a{}_m \end{pmatrix} = -2(\hat{\eta}_{\hat{a}} \sigma_m \hat{\eta}^{\hat{b}}). \quad (4.8)$$

As a result variation of the  $\mathbb{CP}^3$  components of the supervielbein acquires the form

$$\delta_a \Omega_a{}^4(d) = -\bar{j}_{am} da^m, \quad \delta_a \Omega_4{}^a(d) = -j^a{}_m da^m, \quad (4.9)$$

where we have adopted the following definition of the  $SU(3)$  vector dual to a rank 2 tensor that can also carry other indices  $\Sigma$

$$(*j^a)^\Sigma = \frac{1}{2} \varepsilon^{abc} j_{bc}{}^\Sigma, \quad (*j_a)^\Sigma = \frac{1}{2} \varepsilon_{abc} j^{bc\Sigma} \quad (4.10)$$

or simply  $*j^{a\Sigma}$  and  $*j_a{}^\Sigma$  if  $\Sigma$  does not contain  $SU(3)$  indices. Variation of the fermionic supervielbein components associated with the Poincare supersymmetry under localized boundary space-time translations reads

$$\delta_a \hat{\omega}_{\hat{a}}{}^\mu(d) = j_{\hat{a}m}{}^\mu da^m \quad (4.11)$$

with the current contribution

$$j_{\hat{a}m}{}^\mu = \frac{\partial \hat{\omega}_{\hat{a}}{}^\mu(\delta_a)}{\partial a^m} = -e^{-\varphi} \tilde{\sigma}_m^{\mu\nu} \hat{\eta}_{\nu\hat{a}}. \quad (4.12)$$

Then for the variation of remaining fermionic supervielbein components associated with the conformal supersymmetry we obtain

$$\delta_a \hat{\chi}_{\mu\hat{a}}(d) = J_{\mu\hat{a}m} da^m, \quad (4.13)$$

where

$$J_{\mu\hat{a}m} = \frac{\partial \hat{\chi}_{\mu\hat{a}}(\delta_a)}{\partial a^m} = -ie^{2\varphi} (\bar{\eta}\eta) \varepsilon_{\mu\nu} j_{\hat{a}m}{}^\nu \quad (4.14)$$

is the current contribution.

The current density related to the superstring action (2.5) invariance under  $D = 3$  Minkowski space-time translations has the form

$$\mathcal{J}_m^i(\tau, \sigma) = \mathcal{J}_{AdS_m}^i + \mathcal{J}_{CP_m}^i + \mathcal{J}_{WZ_m}^i. \quad (4.15)$$

The  $AdS$  part of the current density

$$\mathcal{J}_{AdS_m}^i = -\frac{1}{4}\sqrt{-g}g^{ij}(\hat{\omega}_{jn} + \hat{c}_{jn})j_m^n \quad (4.16)$$

is determined by the current contribution (4.6), the  $\mathbb{CP}^3$  part

$$\mathcal{J}_{CP_m}^i = \frac{1}{2}\sqrt{-g}g^{ij}(\Omega_{ja}{}^4 * j_m^a + \Omega_{j4}{}^a * \bar{j}_{am}) \quad (4.17)$$

is contributed by Eq.(4.9), and the current contributions (4.12), (4.14) determine the WZ part of the current density

$$\mathcal{J}_{WZ_m}^i = \frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{a}}{}^{\hat{b}}\left(\hat{\omega}_j^{\mu\hat{a}}\varepsilon_{\mu\nu}j_{bm}^\nu + \hat{\chi}_{j\mu}^{\hat{a}}\varepsilon^{\mu\nu}J_{\nu bm}\right). \quad (4.18)$$

#### 4.1.2 Conformal boosts

Transformation properties of the Cartan forms that enter the  $AdS$  part of the  $OSp(4|6)/(SO(1,3) \times U(3))$  superstring action (2.5) under local conformal boosts follow from the expressions (3.21) appropriately restricted

$$\begin{aligned} \delta_b \hat{\omega}^m(d) + \delta_b \hat{c}^m(d) &= j^{mn}db_n + (\hat{l}|_b)^{mn}(\hat{\omega}_n(d) + \hat{c}_n(d)) + 4(\hat{b}|_b)^m \Delta(d), \\ \delta_b \Delta(d) &= j^m db_m - (\hat{b}|_b)^m(\hat{\omega}_m(d) + \hat{c}_m(d)) \end{aligned} \quad (4.19)$$

modulo the current contributions

$$\begin{aligned} j^{mn} &= \frac{\partial(\hat{\omega}^m(\delta_b) + \hat{c}^m(\delta_b))}{\partial b_n} = e^{-2\varphi} A \left\{ (x^2 + (\bar{\theta}\theta)^2)\eta^{mn} - 2x^m x^n + i \left[ (\theta_a \sigma^m \hat{x} \sigma^n \bar{\theta}^a) + (\bar{\theta}^a \sigma^m \hat{x} \sigma^n \theta_a) \right] \right\} \\ &+ A \frac{\partial \hat{b}^m}{\partial b_n} + i e^{2\varphi} \left[ (\eta_a \hat{x} \sigma^n \tilde{\sigma}^m \bar{\eta}^a) + (\bar{\eta}^a \hat{x} \sigma^n \tilde{\sigma}^m \eta_a) + (\eta_a \tilde{\sigma}^m \Lambda_- \sigma^n \bar{\theta}^a) + (\bar{\eta}^a \tilde{\sigma}^m \Lambda_- \sigma^n \theta_a) \right] \\ &+ 2e^{2\varphi}(\bar{\eta}\eta) \left[ (\eta_a \sigma^m \hat{x} \sigma^n \bar{\theta}^a) + (\bar{\eta}^a \sigma^m \hat{x} \sigma^n \theta_a) \right], \end{aligned} \quad (4.20)$$

where we have introduced the following 3d spin-tensors  $\Lambda_{\pm\mu}{}^\nu = \delta_\mu^\nu \pm 2i(\bar{\eta}_\mu^a \theta_a^\nu + \eta_{\mu a} \bar{\theta}^{\nu a})$  that will appear to be useful below, and

$$j^m = \frac{\partial \Delta(\delta_b)}{\partial b_m} = -x^m - i \left[ (\eta_a \hat{x} \sigma^m \bar{\theta}^a) + (\bar{\eta}^a \hat{x} \sigma^m \theta_a) \right]. \quad (4.21)$$

Variation of the  $su(4)$  Cartan forms is obtained by specializing to the conformal boost parameter dependence in Eq.(3.23)

$$\delta_b \Omega_{\hat{a}}{}^{\hat{b}}(d) = \hat{J}_{\hat{a}}{}^{\hat{b}m} db_m + i \left( \Omega_{\hat{a}}{}^{\hat{c}}(d) (\widehat{W}|_b)_{\hat{c}}{}^{\hat{b}} - (\widehat{W}|_b)_{\hat{a}}{}^{\hat{b}} \Omega_{\hat{b}}{}^{\hat{c}}(d) \right) - d(\widehat{W}|_b)_{\hat{a}}{}^{\hat{b}}. \quad (4.22)$$

The current contribution matrix

$$\hat{J}_{\hat{a}}{}^{\hat{b}m} = \frac{\partial}{\partial b_m} \Omega_{\hat{a}}{}^{\hat{b}}(\delta_b) = \begin{pmatrix} \hat{J}_a{}^{bm} & \hat{J}_{ab}{}^m \\ -\hat{J}^{abm} & -\hat{J}_b{}^{am} \end{pmatrix} \quad (4.23)$$

is obtained by  $T$ -transforming the matrix

$$\begin{aligned} J_{\hat{a}}^{\hat{b}m} = & \frac{\partial}{\partial b_m}((\widehat{W}|_b)_{\hat{a}}^{\hat{b}} + \Psi_{\hat{a}}^{\hat{b}}(\delta_b)) = -2 \left[ (\theta_{\hat{a}} \sigma^m \theta^{\hat{b}}) + (x^2 + (\bar{\theta}\theta)^2)(\eta_{\hat{a}} \tilde{\sigma}^m \eta^{\hat{b}}) \right. \\ & \left. - 2x^m(\eta_{\hat{a}} \tilde{x} \eta^{\hat{b}}) - (\eta_{\hat{a}} \hat{x} \sigma^m Z^{\hat{b}}) + (\eta^{\hat{b}} \hat{x} \sigma^m Z_{\hat{a}}) \right], \end{aligned} \quad (4.24)$$

where  $Z_{\hat{a}}^{\mu} = \theta_{\hat{a}}^{\mu} - i(\bar{\theta}\theta)\eta_{\hat{a}}^{\mu}$ . The final form of the current contribution matrix (4.23) is

$$\begin{aligned} \hat{J}_{\hat{a}}^{\hat{b}m} = & (T J^m \bar{T})_{\hat{a}}^{\hat{b}} = -2 \left[ (\hat{\theta}_{\hat{a}} \sigma^m \hat{\theta}^{\hat{b}}) + (x^2 + (\bar{\theta}\theta)^2)(\hat{\eta}_{\hat{a}} \tilde{\sigma}^m \hat{\eta}^{\hat{b}}) \right. \\ & \left. - 2x^m(\hat{\eta}_{\hat{a}} \tilde{x} \hat{\eta}^{\hat{b}}) - (\hat{\eta}_{\hat{a}} \hat{x} \sigma^m \hat{Z}^{\hat{b}}) + (\hat{\eta}^{\hat{b}} \hat{x} \sigma^m \hat{Z}_{\hat{a}}) \right]. \end{aligned} \quad (4.25)$$

Thus variation of the supervielbein components tangent to the  $\mathbb{CP}^3$  manifold is brought to the form

$$\delta_b \Omega_a^{\phantom{a}4}(d) = -\hat{j}_a^{\phantom{a}m} db_m + i(\hat{w}|_b)_b^{\phantom{b}b} \Omega_a^{\phantom{a}4}(d) - i(\hat{w}|_b)_a^{\phantom{a}b} \Omega_b^{\phantom{b}4}(d) \quad (4.26)$$

and c.c. The expressions for the variation of fermionic 1-forms follow from (3.26) and (3.27). Namely, for Cartan forms related to Poincare supersymmetry one derives that

$$\delta_b \hat{\omega}_{\hat{a}}^{\mu}(d) = j_{\hat{a}}^{\mu m} db_m + \frac{1}{4} \hat{\omega}_{\hat{a}}^{\nu}(d) (\hat{l}|_b)_{\nu}^{\mu} + (\hat{b}|_b)^{\mu\nu} \hat{\chi}_{\nu\hat{a}}(d) - i(\widehat{W}|_b)_{\hat{a}}^{\hat{b}} \hat{\omega}_{\hat{b}}^{\mu}(d), \quad (4.27)$$

where the current contribution reads

$$j_{\hat{a}}^{\mu m} = \frac{\partial \hat{\omega}_{\hat{a}}^{\mu}(\delta_b)}{\partial b_m} = e^{-\varphi} \left[ \hat{x}^{\mu\nu} \sigma_{\nu\lambda}^m \hat{Z}_{\hat{a}}^{\lambda} - (x^2 + (\bar{\theta}\theta)^2) \tilde{\sigma}^{m\mu\nu} \hat{\eta}_{\nu\hat{a}} + 2x^m \tilde{x}^{\mu\nu} \hat{\eta}_{\nu\hat{a}} + i(\bar{\theta}\theta) \varepsilon^{\mu\nu} \hat{\eta}_{\rho\hat{a}} \hat{x}^{\rho\lambda} \sigma_{\lambda\nu}^m \right]. \quad (4.28)$$

Correspondingly for Cartan forms related to conformal supersymmetry we find

$$\delta_b \hat{\chi}_{\mu\hat{a}}(d) = J_{\mu\hat{a}}^{\phantom{\mu\hat{a}}m} db_m - \frac{1}{4} (\hat{l}|_b)_{\mu}^{\nu} \hat{\chi}_{\nu\hat{a}}(d) + (\hat{b}|_b)_{\mu\nu} \hat{\omega}_{\hat{a}}^{\nu}(d) - i(\widehat{W}|_b)_{\hat{a}}^{\hat{b}} \hat{\chi}_{\mu\hat{b}}(d) \quad (4.29)$$

with the current contribution

$$J_{\mu\hat{a}}^{\phantom{\mu\hat{a}}m} = \frac{\partial \hat{\chi}_{\mu\hat{a}}(\delta_b)}{\partial b_m} = e^{\varphi} \left( \Lambda_{-\mu}^{\phantom{-\mu}\nu} \sigma_{\nu\lambda}^m \hat{\theta}_{\hat{a}}^{\lambda} - \hat{\eta}_{\nu\hat{a}} \hat{x}^{\nu\lambda} \sigma_{\lambda\rho}^m \Lambda_{-\mu}^{\phantom{-\mu}\rho} - i e^{\varphi} (\bar{\eta}\eta) \varepsilon_{\mu\nu} j_{\hat{a}}^{\nu m} \right). \quad (4.30)$$

The substitution of Eqs.(4.19), (4.26), (4.27) and (4.29) into the superstring action variation under  $D = 3$  conformal boosts yields the current density

$$\mathcal{J}^{im}(\tau, \sigma) = \mathcal{J}_{AdS}^{im} + \mathcal{J}_{CP}^{im} + \mathcal{J}_{WZ}^{im}, \quad (4.31)$$

where

$$\begin{aligned} \mathcal{J}_{AdS}^{im} = & -\sqrt{-g} g^{ij} \left( \frac{1}{4} (\hat{\omega}_{jn} + \hat{c}_{jn}) j^{nm} + \Delta_j j^m \right), \\ \mathcal{J}_{CP}^{im} = & \frac{1}{2} \sqrt{-g} g^{ij} \left( \Omega_{ja}^{\phantom{ja}4} \hat{j}^{am} + \Omega_{j4}^{\phantom{j4}a} \hat{j}_a^{\phantom{a}m} \right), \\ \mathcal{J}_{WZ}^{im} = & \frac{i}{4} \varepsilon^{ij} \mathfrak{J}_{\hat{a}}^{\phantom{\hat{a}}b} \left( \hat{\omega}_j^{\mu\hat{a}} \varepsilon_{\mu\nu} j_{\hat{b}}^{\nu m} + \hat{\chi}_{j\mu}^{\hat{a}} \varepsilon^{\mu\nu} J_{\nu\hat{b}}^{\phantom{\nu\hat{b}}m} \right). \end{aligned} \quad (4.32)$$

### 4.1.3 Dilatations

$osp(4|6)/(so(1,3) \times u(3))$  Cartan forms identified with the  $(10|24)$ -supervielbein components are invariant under the global scale transformations due to the presence of appropriate exponents of the  $AdS_4$  bulk coordinate  $\varphi$ . Hence their variation under coordinate-dependent

scale transformations is determined by the current contributions. In particular, for the components of supervielbein tangent to the  $AdS_4$  space we obtain that

$$\delta_f \hat{\omega}^m(d) + \delta_f \hat{c}^m(d) = j^m df, \quad \delta_f \Delta(d) = j df, \quad (4.33)$$

where

$$\begin{aligned} j^m &= \frac{\partial(\hat{\omega}^m(\delta_f) + \hat{c}^m(\delta_f))}{\partial f} = 2e^{-2\varphi} A x^m + 2e^{2\varphi} (\bar{\eta}\eta) [(\eta_a \sigma^m \bar{\theta}^a) + (\bar{\eta}^a \sigma^m \theta_a)], \\ j &= \frac{\partial \Delta(\delta_f)}{\partial f} = 1 + i(\theta_a^\mu \bar{\eta}_\mu^a + \bar{\theta}^{\mu a} \eta_{\mu a}). \end{aligned} \quad (4.34)$$

$su(4)$  Cartan forms are also scale-invariant

$$\delta_f \Omega_{\hat{a}}^{\hat{b}} = J_{\hat{a}}^{\hat{b}} df \quad (4.35)$$

modulo the current contribution matrix

$$J_{\hat{a}}^{\hat{b}} = \frac{\partial}{\partial f} \Omega_{\hat{a}}^{\hat{b}}(\delta_f) = \begin{pmatrix} j_a^{\hat{b}} & j_{ab} \\ -\bar{j}^{ab} & -j_b^{\hat{a}} \end{pmatrix} = 2 \left( \hat{\Theta}_{\hat{a}}^\mu \hat{\eta}_\mu^{\hat{b}} - \hat{\Theta}^{\mu \hat{b}} \hat{\eta}_{\mu \hat{a}} \right), \quad (4.36)$$

where  $\Theta_{\hat{b}}^\mu = \theta_{\hat{b}}^\mu - \eta_{\nu \hat{b}} \hat{x}^{\nu \mu}$ . So that the variation of  $\mathbb{CP}^3$  part of the supervielbein is governed by the appropriate components of the matrix (4.36)

$$\delta_f \Omega_a^{\hat{4}}(d) = -\bar{j}_a df, \quad \delta_f \Omega_{\hat{4}}^a(d) = -j^a df. \quad (4.37)$$

Fermionic supervielbein components related to Poincare supersymmetry obey the transformation rules

$$\delta_f \hat{\omega}_a^\mu(d) = j_a^\mu df \quad (4.38)$$

with the current contribution

$$j_a^\mu = \frac{\partial \hat{\omega}_a^\mu(\delta_f)}{\partial f} = e^{-\varphi} (\hat{\Theta}_{\hat{a}}^\mu - 2\tilde{x}^{\mu\nu} \hat{\eta}_{\nu \hat{a}}), \quad (4.39)$$

while those related to conformal supersymmetry transform as

$$\delta_f \hat{\chi}_{\mu \hat{a}}(d) = J_{\mu \hat{a}} df, \quad (4.40)$$

where the corresponding current contribution is given by

$$J_{\mu \hat{a}} = \frac{\partial \hat{\chi}_{\mu \hat{a}}(\delta_f)}{\partial f} = -e^\varphi (\Lambda_{-\mu}^{\nu} \hat{\eta}_{\nu \hat{a}} + i e^\varphi (\bar{\eta}\eta) \varepsilon_{\mu\nu} j_{\hat{a}}^\nu). \quad (4.41)$$

Above presented current contributions determine the Noether current density related to the scale invariance of superstring action (2.5)

$$\mathcal{J}^i(\tau, \sigma) = \mathcal{J}_{AdS}^i + \mathcal{J}_{CP}^i + \mathcal{J}_{WZ}^i. \quad (4.42)$$

Specifically the  $AdS$  part of the current density

$$\mathcal{J}_{AdS}^i = -\sqrt{-g} g^{ij} \left( \frac{1}{4} (\hat{\omega}_{jm} + \hat{c}_{jm}) j^m + \Delta_j j \right) \quad (4.43)$$

is contributed by Eq.(4.34), the  $\mathbb{CP}^3$  part

$$\mathcal{J}_{CP}^i = \frac{1}{2} \sqrt{-g} g^{ij} (\Omega_{j\hat{a}}^{\hat{4}} j^{\hat{a}} + \Omega_{j\hat{4}}^{\hat{a}} \bar{j}_{\hat{a}}) \quad (4.44)$$

is determined by the current contributions that enter (4.37), and the WZ part

$$\mathcal{J}_{WZ}^i = \frac{i}{4} \varepsilon^{ij} \mathfrak{J}_{\hat{a}}^{\hat{b}} \left( \hat{\omega}_j^{\mu \hat{a}} \varepsilon_{\mu\nu} j_b^\nu + \hat{\chi}_{j\mu}^{\hat{a}} \varepsilon^{\mu\nu} J_{\nu \hat{b}} \right) \quad (4.45)$$

receives contributions from (4.39) and (4.41).

#### 4.1.4 Lorentz rotations

Under local  $SO(1, 2)$  Lorentz rotations with parameters  $l^{mn}$  Cartan forms identified with the supervielbein components tangent to the  $AdS_4$  space exhibit the following transformation properties

$$\begin{aligned}\delta_l \hat{\omega}^m(d) + \delta_l \hat{c}^m(d) &= j^m_{kn} dl^{kn} + l^{mn} (\hat{\omega}_n(d) + \hat{c}_n(d)), \\ \delta_l \Delta(d) &= j_{mn} dl^{mn}\end{aligned}\tag{4.46}$$

with the current contributions

$$\begin{aligned}j^m_{kn} &= \frac{\partial(\hat{\omega}^m(\delta_l) + \hat{c}^m(\delta_l))}{\partial l^{kn}} = \frac{1}{2} e^{-2\varphi} A \left( \delta_k^m x_n - \delta_n^m x_k + i(\bar{\theta}\theta) \varepsilon^m_{kn} \right) \\ &+ \frac{1}{2} e^{2\varphi} (\bar{\eta}\eta) \left\{ \delta_k^m \left[ (\eta_a \sigma_n \bar{\theta}^a) + (\bar{\eta}^a \sigma_n \theta_a) \right] - (k \leftrightarrow n) + \varepsilon^m_{kn} \left[ i + (\bar{\eta}\theta) + (\bar{\theta}\eta) \right] \right\}, \\ j_{mn} &= \frac{\partial \Delta(\delta_l)}{\partial l^{mn}} = \frac{i}{4} \left[ (\theta_a \sigma_{mn} \bar{\eta}^a) + (\bar{\theta}^a \sigma_{mn} \eta_a) \right].\end{aligned}\tag{4.47}$$

The  $su(4)$  Cartan forms are obviously  $D = 3$  Lorentz invariant

$$\delta_l \Omega_{\hat{a}}^{\hat{b}}(d) = J_{\hat{a}}^{\hat{b}}{}_{mn} dl^{mn}\tag{4.48}$$

modulo the current contribution matrix

$$J_{\hat{a}}^{\hat{b}}{}_{mn} = \frac{\partial}{\partial l^{mn}} \Omega_{\hat{a}}^{\hat{b}}(\delta_l) = \begin{pmatrix} j_{\hat{a}}^{\hat{b}}{}_{mn} & j_{abmn} \\ -\bar{j}^{ab}{}_{mn} & -j_{\hat{b}}^{\hat{a}}{}_{mn} \end{pmatrix} = \frac{1}{2} \left[ (\hat{\Theta}_{\hat{a}} \sigma_{mn} \hat{\eta}^{\hat{b}}) - (\hat{\Theta}^{\hat{b}} \sigma_{mn} \hat{\eta}_{\hat{a}}) \right].\tag{4.49}$$

Hence variation of the supervielbein components tangent to the  $\mathbb{CP}^3$  manifold is extracted from (4.49)

$$\delta_l \Omega_a^{\hat{b}}(d) = -\bar{j}_{amnn}^{\hat{b}} dl^{mn}, \quad \delta_l \Omega_{\hat{a}}^a(d) = -j_{\hat{a}mn}^a dl^{mn}.\tag{4.50}$$

Variation of the supervielbein fermionic components that are identified with the Cartan forms related to Poincare supersymmetry follows from the general expression (3.26)

$$\delta_l \hat{\omega}_{\hat{a}}^{\mu}(d) = j_{\hat{a}mn}^{\mu} dl^{mn} + \frac{1}{4} \hat{\omega}_{\hat{a}}^{\nu}(d) l_{\nu}^{\mu},\tag{4.51}$$

where the current contribution reads

$$j_{\hat{a}mn}^{\mu} = \frac{\partial \hat{\omega}_{\hat{a}}^{\mu}(\delta_l)}{\partial l^{mn}} = \frac{1}{4} e^{-\varphi} \left( \hat{\Theta}_{\hat{a}}^{\nu} \sigma_{mn\nu}^{\mu} - \hat{x}^{\mu\lambda} \sigma_{mn\lambda}^{\nu} \hat{\eta}_{\nu\hat{a}} \right).\tag{4.52}$$

Transformation properties of the Cartan forms related to conformal supersymmetry identified with another half supervielbein fermionic components follow from Eq.(3.27)

$$\delta_l \hat{\chi}_{\mu\hat{a}}(d) = J_{\mu\hat{a}mn} dl^{mn} - \frac{1}{4} l_{\mu}^{\nu} \hat{\chi}_{\nu\hat{a}}(d)\tag{4.53}$$

with the current contribution

$$J_{\mu\hat{a}mn} = \frac{\partial \hat{\chi}_{\mu\hat{a}}(\delta_l)}{\partial l^{mn}} = -e^{\varphi} \left( \frac{1}{4} \Lambda_{-}^{\nu} \sigma_{mn\nu}^{\lambda} \hat{\eta}_{\lambda\hat{a}} + i e^{\varphi} (\bar{\eta}\eta) \varepsilon_{\mu\nu} j_{\hat{a}mn}^{\nu} \right).\tag{4.54}$$

Then one is able to derive the current density related to global  $SO(1, 2)$  symmetry of the superstring action (2.5)

$$\mathcal{J}_{mn}^i(\tau, \sigma) = \mathcal{J}_{AdS}^i{}_{mn} + \mathcal{J}_{CP}^i{}_{mn} + \mathcal{J}_{WZ}^i{}_{mn}.\tag{4.55}$$

The form of the  $AdS$  part of the current density

$$\mathcal{J}_{AdS}{}^i{}_{mn} = -\sqrt{-g}g^{ij} \left( \frac{1}{4}(\hat{\omega}_{jl} + \hat{c}_{jl})j^l{}_{mn} + \Delta_j j_{mn} \right), \quad (4.56)$$

is determined by Eq.(4.47), that of the  $\mathbb{CP}^3$  part

$$\mathcal{J}_{CP}{}^i{}_{mn} = \frac{1}{2}\sqrt{-g}g^{ij} (\Omega_{ja}{}^4 * j^a{}_{mn} + \Omega_{j4}{}^a * \bar{j}_{amn}) \quad (4.57)$$

by Eq.(4.50), and the WZ part of the current density

$$\mathcal{J}_{WZ}{}^i{}_{mn} = \frac{i}{4}\varepsilon^{ij} \mathfrak{J}_{\hat{a}}{}^{\hat{b}} \left( \hat{\omega}_j^{\mu\hat{a}} \varepsilon_{\mu\nu} j_{\hat{b}mn}^\nu + \hat{\chi}_{j\mu}^{\hat{a}} \varepsilon^{\mu\nu} J_{\nu\hat{b}mn} \right) \quad (4.58)$$

is contributed by Eqs.(4.52) and (4.54).

## 4.2 Noether currents associated with $SU(4)$ $R$ -symmetry

### 4.2.1 $U(3)$ Rotations

Global  $U(3)$  rotations represent an obvious symmetry of the  $AdS_4$  part of (10|24)-supervielbein thus the nontrivial part of its variation under the coordinate-dependent  $U(3)$  rotations

$$\delta_w \hat{\omega}^m(d) + \delta_w \hat{c}^m(d) = j^m{}_a{}^b dw_b{}^a, \quad \delta_w \Delta = j_a{}^b dw_b{}^a \quad (4.59)$$

is concentrated in the current contributions

$$\begin{aligned} j^m{}_a{}^b &= \frac{\partial(\hat{\omega}^m(\delta_w) + \hat{c}^m(\delta_w))}{\partial w_b{}^a} = 2e^{-2\varphi} A [\delta_a^b (\theta_c \sigma^m \bar{\theta}^c) - (\theta_a \sigma^m \bar{\theta}^b)] + 2e^{2\varphi} \delta_a^b \{ (\eta_c \tilde{\sigma}^m \bar{\eta}^c) \\ &\quad - i(\bar{\eta}\eta) [(\theta_c \sigma^m \bar{\eta}^c) + (\eta_c \sigma^m \bar{\theta}^c)] \} - 2e^{2\varphi} \{ (\eta_a \tilde{\sigma}^m \bar{\eta}^b) - i(\bar{\eta}\eta) [(\theta_a \sigma^m \bar{\eta}^b) + (\eta_a \sigma^m \bar{\theta}^b)] \}, \\ j_a{}^b &= \frac{\partial \Delta(\delta_w)}{\partial w_b{}^a} = \delta_a^b (\bar{\theta}^{\mu c} \eta_{\mu c} + \bar{\eta}_\mu^c \theta_c^\mu) + \theta_a^\mu \bar{\eta}_\mu^b + \eta_{\mu a} \bar{\theta}^{\mu b}. \end{aligned} \quad (4.60)$$

Transformation properties of the  $su(4)$  Cartan forms under  $U(3)$  rotations are derived from Eq.(3.23)

$$\delta_w \Omega_{\hat{c}}{}^{\hat{d}}(d) = \hat{J}_{\hat{c}}{}^{\hat{d}}{}_a{}^b dw_b{}^a + i(\Omega_{\hat{c}}{}^{\hat{e}}(d)(\widehat{W}|_w)_{\hat{e}}{}^{\hat{d}} - (\widehat{W}|_w)_{\hat{c}}{}^{\hat{e}} \Omega_{\hat{e}}{}^{\hat{d}}(d)) - d(\widehat{W}|_w)_{\hat{c}}{}^{\hat{d}}. \quad (4.61)$$

The current contribution matrix

$$\hat{J}_{\hat{c}}{}^{\hat{d}}{}_a{}^b = \frac{\partial}{\partial w_b{}^a} \Omega_{\hat{c}}{}^{\hat{d}}(\delta_w) = \begin{pmatrix} \hat{J}_{\hat{c}}{}^{\hat{d}}{}_a{}^b & \hat{J}_{\hat{c}\hat{d}a}{}^b \\ -\hat{J}^{\hat{c}\hat{d}}{}_a{}^b & -\hat{J}_{\hat{d}}{}^{\hat{c}}{}_a{}^b \end{pmatrix} \quad (4.62)$$

is obtained by  $T$ -transformation of the matrix

$$\begin{aligned} J_{\hat{c}}{}^{\hat{d}}{}_a{}^b &= \frac{\partial}{\partial w_b{}^a} ((\widehat{W}|_w)_{\hat{c}}{}^{\hat{d}} + \Psi_{\hat{c}}{}^{\hat{d}}(\delta_w)) = \delta_{\hat{c}}{}^{\hat{b}} \delta_a{}^{\hat{d}} - \delta_{\hat{c}\hat{a}} \delta^{b\hat{d}} + 4\eta_{\mu\hat{c}} (\theta_a^\mu \bar{\theta}^{\nu b} + \theta_a^\nu \bar{\theta}^{\mu b}) \eta_\nu^{\hat{d}} \\ &\quad + \delta_a^b \left[ \frac{i}{2} \mathfrak{J}_{\hat{c}}{}^{\hat{d}} + (\mathfrak{J}\theta^\mu)_{\hat{c}} \eta_\mu^{\hat{d}} - (\mathfrak{J}\theta^\mu)^{\hat{d}} \eta_{\mu\hat{c}} - 4\eta_{\mu\hat{c}} (\theta_e^\mu \bar{\theta}^{\nu e} + \theta_e^\nu \bar{\theta}^{\mu e}) \eta_\nu^{\hat{d}} \right] \\ &\quad + 2i \left[ \theta_a^\mu (\eta_{\mu\hat{c}} \delta^{b\hat{d}} - \eta_\mu^{\hat{d}} \delta_{\hat{c}}{}^b) + \bar{\theta}^{\mu b} (\eta_\mu^{\hat{d}} \delta_{\hat{c}a} - \eta_{\mu\hat{c}} \delta_a{}^{\hat{d}}) \right], \end{aligned} \quad (4.63)$$

where the following objects have been introduced

$$\delta_{\hat{c}}{}^b = \begin{pmatrix} \delta_c^b \\ 0 \end{pmatrix}, \quad \delta_{\hat{c}a} = \begin{pmatrix} 0 \\ \delta_a^c \end{pmatrix}, \quad \delta_a{}^{\hat{d}} = (\delta_a^d \ 0), \quad \delta^{b\hat{d}} = (0 \ \delta_d^b). \quad (4.64)$$

So that the explicit form of the current contribution matrix is

$$\hat{J}_{\hat{c}}^{\hat{d} \ a \ b} = (T J_a^{\ b} \bar{T})_{\hat{c}}^{\hat{d}} = \mathcal{T}_{\hat{c}}^b \mathcal{T}_{\hat{a}}^{\hat{d}} - \mathcal{T}_{\hat{c}\hat{a}} \mathcal{T}^{\hat{d}b} + \frac{i}{2} \delta_a^b (\mathcal{T} \mathfrak{J} \bar{T})_{\hat{c}}^{\hat{d}}. \quad (4.65)$$

The matrix  $\mathcal{T}_{\hat{a}}^{\hat{b}}$  equal

$$\mathcal{T}_{\hat{a}}^{\hat{b}} = \begin{pmatrix} \mathcal{T}_a^b & \mathcal{T}_{ab} \\ \mathcal{T}^a_b & \mathcal{T}_{\hat{a}}^{\hat{b}} \end{pmatrix} = T_{\hat{a}}^{\hat{b}} + 2i\hat{\eta}_{\nu\hat{a}}\theta^{\nu\hat{b}} \quad (4.66)$$

will also be used below. From Eqs.(4.61) and (4.65) one derives transformation properties of the  $\mathbb{CP}^3$  part of the bosonic supervielbein

$$\delta_w \Omega_c^{\ 4}(d) = -(\hat{*}\hat{j}_c)_a^{\ b} dw_b^{\ a} + i(\hat{w}|_w)_d^{\ \hat{d}} \Omega_c^{\ 4}(d) - i(\hat{w}|_w)_c^{\ \hat{c}} \Omega_d^{\ 4}(d) \quad (4.67)$$

and c.c. expression.

Fermionic supervielbein components associated with the Poincare supersymmetry transform under local  $U(3)$  rotations as follows

$$\delta_w \hat{\omega}_{\hat{c}}^{\mu}(d) = j_{\hat{c} \ a}^{\mu \ b} dw_b^{\ a} - i(\widehat{W}|_w)_{\hat{c}}^{\hat{d}} \hat{\omega}_{\hat{d}}^{\mu}(d), \quad (4.68)$$

where

$$j_{\hat{c} \ a}^{\mu \ b} = \frac{\partial \hat{\omega}_{\hat{c}}^{\mu}(\delta_w)}{\partial w_b^{\ a}} = e^{-\varphi} \left[ \frac{1}{2} \delta_a^b (\mathcal{T} \mathfrak{J} \theta^{\mu})_{\hat{c}} - i\theta_a^{\mu} \mathcal{T}_{\hat{c}}^b + i\bar{\theta}^{\mu b} \mathcal{T}_{\hat{c}\hat{a}} \right] \quad (4.69)$$

represents the current contribution. Analogously variation of the fermionic supervielbein components related to conformal supersymmetry is given by the expression

$$\delta_w \hat{\chi}_{\mu\hat{c}}(d) = J_{\mu\hat{c}a}^{\ b} dw_b^{\ a} - i(\widehat{W}|_w)_{\hat{c}}^{\hat{d}} \hat{\chi}_{\mu\hat{d}}(d) \quad (4.70)$$

with the current contribution

$$J_{\mu\hat{c}a}^{\ b} = \frac{\partial \hat{\chi}_{\mu\hat{c}}(\delta_w)}{\partial w_b^{\ a}} = e^{\varphi} \left[ \frac{1}{2} \delta_a^b (\mathcal{T} \mathfrak{J} \eta_{\mu})_{\hat{c}} - i\eta_{\mu a} \mathcal{T}_{\hat{c}}^b + i\bar{\eta}_{\mu}^b \mathcal{T}_{\hat{c}\hat{a}} + ie^{\varphi} (\bar{\eta} \eta) \varepsilon_{\mu\nu} j_{\hat{c} \ a}^{\nu \ b} \right]. \quad (4.71)$$

In summary the current density associated with the  $U(3)$  global invariance of superstring action (2.5)

$$\mathcal{J}_a^{\ i \ b}(\tau, \sigma) = \mathcal{J}_{AdS}^{\ i \ b} + \mathcal{J}_{CP}^{\ i \ b} + \mathcal{J}_{WZ}^{\ i \ b} \quad (4.72)$$

consists of three summands

$$\mathcal{J}_{AdS}^{\ i \ b} = -\sqrt{-g} g^{ij} \left( \frac{1}{4} (\hat{\omega}_{jm} + \hat{c}_{jm}) j_a^{\ m \ b} + \Delta_j j_a^{\ b} \right), \quad (4.73)$$

$$\mathcal{J}_{CP}^{\ i \ b} = \frac{1}{2} \sqrt{-g} g^{ij} \left( \Omega_{jc}^{\ 4} (\hat{*}\hat{j}^c)_a^{\ b} + \Omega_{j4}^{\ c} (\hat{*}\hat{j}_c)_a^{\ b} \right), \quad (4.74)$$

and

$$\mathcal{J}_{WZ}^{\ i \ b} = \frac{i}{4} \varepsilon^{ij} \mathfrak{J}_{\hat{c}}^{\hat{d}} \left( \hat{\omega}_{\hat{j}}^{\mu\hat{c}} \varepsilon_{\mu\nu} j_{\hat{d} \ a}^{\nu \ b} + \hat{\chi}_{\hat{j}\mu}^{\hat{c}} \varepsilon^{\mu\nu} J_{\nu\hat{d}a}^{\ b} \right) \quad (4.75)$$

that are determined by the current contributions of the  $osp(4|6)/(so(1,3) \times u(3))$  Cartan forms (4.60), (4.67), (4.69) and (4.71).

### 4.2.2 $SU(4)/U(3)$ transformations

As in the case of  $U(3)$  rotations Cartan forms from the  $AdS_4$  sector are invariant under local  $SU(4)/U(3)$  transformations

$$\delta_y \hat{\omega}^m(d) + \delta_y \hat{c}^m(d) = j^m{}_a dy^a + \bar{j}^{ma} d\bar{y}_a, \quad \delta_y \Delta(d) = j_a dy^a + \bar{j}^a d\bar{y}_a \quad (4.76)$$

modulo the current contributions

$$\begin{aligned} j^m{}_a &= \frac{\partial(\hat{\omega}^m(\delta_y) + \hat{c}^m(\delta_y))}{\partial y^a} = -\varepsilon_{abc} \left\{ e^{-2\varphi} A(\bar{\theta}^b \sigma^m \bar{\theta}^c) + e^{2\varphi} [(\bar{\eta}^b \tilde{\sigma}^m \bar{\eta}^c) - 2i(\bar{\eta}\eta)(\bar{\theta}^b \tilde{\sigma}^m \bar{\eta}^c)] \right\}, \\ j_a &= \frac{\partial \Delta(\delta_y)}{\partial y^a} = \varepsilon_{abc} \bar{\theta}^{\mu b} \bar{\eta}^c_\mu \end{aligned} \quad (4.77)$$

and c.c. Transformation properties of the  $su(4)$  Cartan forms follow from the general formula (3.23) by specializing to  $SU(4)/U(3)$  rotations

$$\delta_y \Omega_{\hat{b}}^{\hat{c}}(d) = J_{\hat{b}}^{\hat{c}}{}_a dy^a + \bar{J}_{\hat{b}}^{\hat{c}a} d\bar{y}_a + i \left( \Omega_{\hat{b}}^{\hat{d}}(d) (\widehat{W}|_y)_{\hat{d}}^{\hat{c}} - (\widehat{W}|_y)_{\hat{b}}^{\hat{d}} \Omega_{\hat{d}}^{\hat{c}}(d) \right) - d(\widehat{W}|_y)_{\hat{b}}^{\hat{c}}. \quad (4.78)$$

Corresponding current contribution matrices equal

$$J_{\hat{b}}^{\hat{c}}{}_a = \frac{\partial}{\partial y^a} \Omega_{\hat{b}}^{\hat{c}}(\delta_y) = \begin{pmatrix} j_b^c{}_a & j_{bca} \\ j^{bc}{}_a & -j_c^b{}_a \end{pmatrix} = -\varepsilon_{ade} \mathcal{T}_{\hat{b}}^d \mathcal{T}^{\hat{c}e} \quad (4.79)$$

and

$$\bar{J}_{\hat{b}}^{\hat{c}a} = \frac{\partial}{\partial \bar{y}_a} \Omega_{\hat{b}}^{\hat{c}}(\delta_y) = \begin{pmatrix} \bar{j}_b^{ca} & -\bar{j}_{bc}^a \\ -\bar{j}^{bca} & -\bar{j}_c^{ba} \end{pmatrix} = \varepsilon^{ade} \mathcal{T}_{\hat{b}d} \mathcal{T}^{\hat{c}e}. \quad (4.80)$$

So that one extracts from the above expressions the transformation rules for the supervielbein bosonic components tangent to the  $\mathbb{CP}^3$  manifold

$$\delta_y \Omega_b^4(d) = (*j_b)_a dy^a - (*\bar{j}_b)^a d\bar{y}_a + i(\hat{w}|_y)_c^b \Omega_b^4(d) - i(\hat{w}|_y)_b^c \Omega_c^4(d). \quad (4.81)$$

Supervielbein fermionic components associated with the Poincare supersymmetry have the following properties under  $SU(4)/U(3)$  transformations

$$\delta_y \hat{\omega}_b^\mu(d) = j_{ba}^\mu dy^a + \bar{j}_b^{\mu a} d\bar{y}_a - i(\widehat{W}|_y)_{\hat{b}}^{\hat{c}} \hat{\omega}_{\hat{c}}^\mu(d) \quad (4.82)$$

with the current contributions

$$j_{ba}^\mu = \frac{\partial \hat{\omega}_b^\mu(\delta_y)}{\partial y^a} = i e^{-\varphi} \varepsilon_{acd} \mathcal{T}_{\hat{b}}^c \bar{\theta}^{\mu d} \quad (4.83)$$

and c.c. Supervielbein fermionic components related to conformal supersymmetry transform as

$$\delta_y \hat{\chi}_{\mu\hat{b}}(d) = J_{\mu\hat{b}a} dy^a + \bar{J}_{\mu\hat{b}}^a d\bar{y}_a - i(\widehat{W}|_y)_{\hat{b}}^{\hat{c}} \hat{\chi}_{\mu\hat{c}}(d), \quad (4.84)$$

where the current contributions can be brought to the form

$$J_{\mu\hat{b}a} = \frac{\partial \hat{\chi}_{\mu\hat{b}}(\delta_y)}{\partial y^a} = i e^\varphi (\varepsilon_{acd} \mathcal{T}_{\hat{b}}^c \bar{\eta}_\mu^d + e^\varphi (\bar{\eta}\eta) \varepsilon_{\mu\nu} j_{ba}^\nu) \quad (4.85)$$

and c.c.



Above derived current contributions of the supervielbein components determine the Noether current density associated with  $SU(4)/U(3)$  global invariance of the superstring action

$$\mathcal{J}_a^i(\tau, \sigma) = \mathcal{J}_{AdS}^i{}_a + \mathcal{J}_{CP}^i{}_a + \mathcal{J}_{WZ}^i{}_a \quad (4.86)$$

and c.c. expression. The summands contributed by Eqs.(4.77), (4.81), (4.83) and (4.85) respectively take the form

$$\mathcal{J}_{AdS}^i{}_a = -\sqrt{-g}g^{ij} \left( \frac{1}{4}(\hat{\omega}_{jm} + \hat{c}_{jm})j^m{}_a + \Delta_j j_a \right), \quad (4.87)$$

$$\mathcal{J}_{CP}^i{}_a = \frac{1}{2}\sqrt{-g}g^{ij} (\Omega_{jb}{}^4 (*j^b)_a - \Omega_{j4}{}^b (*j_b)_a), \quad (4.88)$$

and

$$\mathcal{J}_{WZ}^i{}_a = \frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{b}}{}^{\hat{c}} \left( \hat{\omega}_j^{\mu\hat{b}}\varepsilon_{\mu\nu}j_{\hat{c}a}^\nu + \hat{\chi}_{j\mu}^{\hat{b}}\varepsilon^{\mu\nu}J_{\nu\hat{c}a} \right). \quad (4.89)$$

### 4.3 Noether currents associated with $D = 3 \mathcal{N} = 6$ Poincare supersymmetry

The superstring action (2.5) is manifestly invariant under Poincare supersymmetry as  $D = 3 \mathcal{N} = 6$  supercoordinates  $(x^m, \theta_a^\mu, \bar{\theta}^{\mu a})$  that are the only non-trivially transforming ones enter through the supersymmetric Volkov-Akulov 1-forms [33]. Hence non-invariance of the  $AdS$  part of the supervielbein bosonic components under coordinate-dependent Poincare supersymmetry transformations

$$\delta_\varepsilon \hat{\omega}^m(d) + \delta_\varepsilon \hat{c}^m(d) = j_{\mu}^{ma} d\varepsilon_a^\mu - \bar{j}_{\mu a}^m d\bar{\varepsilon}^{\mu a}, \quad \delta_\varepsilon \Delta(d) = j_{\mu}^a d\varepsilon_a^\mu - \bar{j}_{\mu a} d\bar{\varepsilon}^{\mu a}, \quad (4.90)$$

is accounted by the current contributions which explicit form is

$$\begin{aligned} j_{\mu}^{ma} &= \frac{\partial(\hat{\omega}^m(\delta_\varepsilon) + \hat{c}^m(\delta_\varepsilon))}{\partial \varepsilon_a^\mu} = 2\sigma_{\mu\nu}^m [ie^{-2\varphi} A \bar{\theta}^{\nu a} + e^{2\varphi} (\bar{\eta}\eta) \bar{\eta}^{\nu a}], \\ j_{\mu}^a &= \frac{\partial \Delta(\delta_\varepsilon)}{\partial \varepsilon_a^\mu} = -i\bar{\eta}_\mu^a \end{aligned} \quad (4.91)$$

and c.c. expressions. The  $su(4)$  Cartan forms are also  $D = 3 \mathcal{N} = 6$  super-Poincare invariant

$$\delta_\varepsilon \Omega_{\hat{b}}{}^{\hat{c}}(d) = J_{\hat{b}}{}^{\hat{c}a}{}_{\mu} d\varepsilon_a^\mu - \bar{J}_{\hat{b}}{}^{\hat{c}}{}_{\mu a} d\bar{\varepsilon}^{\mu a} \quad (4.92)$$

modulo the current contribution matrices

$$J_{\hat{b}}{}^{\hat{c}a}{}_{\mu} = \frac{\partial}{\partial \varepsilon_a^\mu} \Omega_{\hat{b}}{}^{\hat{c}}(\delta_\varepsilon) = \begin{pmatrix} j_{\mu}^{ca} & j_{bc\mu}^a \\ j_{bca\mu} & -j_c{}^b{}_{\mu} \end{pmatrix} = 2(\hat{\eta}_{\mu\hat{b}} \mathcal{T}^{\hat{c}a} - \mathcal{T}_{\hat{b}}^a \hat{\eta}_\mu^{\hat{c}}) \quad (4.93)$$

and

$$\bar{J}_{\hat{b}}{}^{\hat{c}}{}_{\mu a} = -\frac{\partial}{\partial \bar{\varepsilon}^{\mu a}} \Omega_{\hat{b}}{}^{\hat{c}}(\delta_\varepsilon) = \begin{pmatrix} \bar{j}_{\hat{b}}{}^{\hat{c}\mu a} & -\bar{j}_{bc\mu a} \\ -\bar{j}_{\hat{b}}{}^{\hat{c}}{}_{\mu a} & -\bar{j}_c{}^b{}_{\mu} \end{pmatrix} = 2(\mathcal{T}_{\hat{b}a} \hat{\eta}_\mu^{\hat{c}} - \hat{\eta}_{\mu\hat{b}} \mathcal{T}^{\hat{c}a}) \quad (4.94)$$

so that the  $\mathbb{CP}^3$  part of the bosonic supervielbein transforms as

$$\delta_\varepsilon \Omega_b{}^4(d) = (*j_b)_\mu^a d\varepsilon_a^\mu + (*\bar{j}_b)_{\mu a} d\bar{\varepsilon}^{\mu a}, \quad \delta_\varepsilon \Omega_4{}^b(d) = -(j^b)_\mu^a d\varepsilon_a^\mu - (\bar{j}^b)_{\mu a} d\bar{\varepsilon}^{\mu a}. \quad (4.95)$$

Cartan forms associated with the Poincare supersymmetry are manifestly  $D = 3 \mathcal{N} = 6$  super-Poincare invariant

$$\delta_\varepsilon \hat{\omega}_b^\nu(d) = j_{\hat{b}}^{\nu a}{}_{\mu} d\varepsilon_a^\mu + \bar{j}_{\hat{b}}{}^{\nu}{}_{\mu a} d\bar{\varepsilon}^{\mu a} \quad (4.96)$$

up to the current contributions

$$j_{\hat{b}\mu}^{\nu a} = \frac{\partial \hat{\omega}_{\hat{b}}^{\nu}(\delta_{\varepsilon})}{\partial \varepsilon_a^{\mu}} = e^{-\varphi} \left( \delta_{\mu}^{\nu} \mathcal{T}_{\hat{b}}^a + 2i\hat{\eta}_{\mu\hat{b}} \bar{\theta}^{\nu a} \right) \quad (4.97)$$

and c.c., as well as Cartan forms associated with the conformal supersymmetry

$$\delta_{\varepsilon} \hat{\chi}_{\nu\hat{b}}(d) = J_{\nu\hat{b}\mu}^a d\varepsilon_a^{\mu} + \bar{J}_{\nu\hat{b}\mu a} d\bar{\varepsilon}^{\mu a} \quad (4.98)$$

with the corresponding current contributions given by

$$J_{\nu\hat{b}\mu}^a = \frac{\partial \hat{\chi}_{\nu\hat{b}}(\delta_{\varepsilon})}{\partial \varepsilon_a^{\mu}} = ie^{\varphi} \left( 2\hat{\eta}_{\mu\hat{b}} \bar{\eta}_{\nu}^a - e^{\varphi} (\bar{\eta}\eta) \varepsilon_{\nu\lambda} j_{\hat{b}\mu}^{\lambda a} \right) \quad (4.99)$$

and c.c.

Putting all together contributions (4.91), (4.95), (4.97) and (4.99) allows to determine the current density related to  $D = 3$   $\mathcal{N} = 6$  supersymmetry invariance of the superstring action (2.5)

$$\mathcal{J}_{\mu}^{ia}(\tau, \sigma) = \mathcal{J}_{AdS\mu}^{ia} + \mathcal{J}_{CP\mu}^{ia} + \mathcal{J}_{WZ\mu}^{ia} \quad (4.100)$$

and the c.c. one. The individual summands entering the current density (4.100) equal

$$\mathcal{J}_{AdS\mu}^{ia} = -\sqrt{-g} g^{ij} \left( \frac{1}{4} (\hat{\omega}_{jm} + \hat{c}_{jm}) j_{\mu}^{ma} + \Delta_j j_{\mu}^a \right), \quad (4.101)$$

$$\mathcal{J}_{CP\mu}^{ia} = \frac{1}{2} \sqrt{-g} g^{ij} \left( \Omega_{jb}{}^4 (*j^b)_{\mu}^a - \Omega_{j4}{}^b (*j_b)_{\mu}^a \right) \quad (4.102)$$

and

$$\mathcal{J}_{WZ\mu}^{ia} = \frac{i}{4} \varepsilon^{ij} \mathfrak{J}_{\hat{b}}^{\hat{c}} \left( \hat{\omega}_j^{\nu\hat{b}} \varepsilon_{\nu\lambda} j_{\hat{c}\mu}^{\lambda a} + \hat{\chi}_{j\nu}^{\hat{b}} \varepsilon^{\nu\lambda} J_{\lambda\hat{c}\mu}^a \right). \quad (4.103)$$

#### 4.4 Noether currents associated with $D = 3$ $\mathcal{N} = 6$ conformal supersymmetry

Transformation properties of the supervielbein bosonic components in the directions tangent to the  $AdS_4$  space can be obtained from the general expressions (3.21) by appropriately restricting the parameters of  $SO(1, 3)$  compensating rotations

$$\begin{aligned} \delta_{\xi} \hat{\omega}^m(d) + \delta_{\xi} \hat{c}^m(d) &= j^{m\mu a} d\xi_{\mu a} - \bar{j}_{\mu}^m d\bar{\xi}_{\mu}^a + (\hat{l}|_{\xi})^{mn} (\hat{\omega}_n(d) + \hat{c}_n(d)) + 4(\hat{b}|_{\xi})^m \Delta(d), \\ \delta_{\xi} \Delta(d) &= j^{\mu a} d\xi_{\mu a} - \bar{j}_{\mu}^{\mu} d\bar{\xi}_{\mu}^a - (\hat{b}|_{\xi})^m (\hat{\omega}_m(d) + \hat{c}_m(d)) \end{aligned} \quad (4.104)$$

and adding the current contribution terms

$$\begin{aligned} j^{m\mu a} &= \frac{\partial (\hat{\omega}^m(\delta_{\xi}) + \hat{c}^m(\delta_{\xi}))}{\partial \xi_{\mu a}} = 2 \left[ ie^{-2\varphi} A \bar{\theta}^{\lambda a} \sigma_{\lambda\nu}^m \hat{x}^{\nu\mu} + ie^{2\varphi} \bar{\eta}_{\lambda}^a \tilde{\sigma}^{m\lambda\nu} \Lambda_{-\nu}{}^{\mu} \right. \\ &\quad \left. + e^{2\varphi} (\bar{\eta}\eta) \left( \bar{\eta}^{\lambda a} \sigma_{\lambda\nu}^m \hat{x}^{\nu\mu} - \bar{\theta}_{\lambda}^a \tilde{\sigma}^{m\lambda\nu} \Lambda_{+\nu}{}^{\mu} \right) \right], \\ j^{\mu a} &= \frac{\partial \Delta(\delta_{\xi})}{\partial \xi_{\mu a}} = -i \left( \bar{\theta}^{\nu a} \Lambda_{-\nu}{}^{\mu} + \bar{\eta}_{\nu}^a \hat{x}^{\nu\mu} \right) \end{aligned} \quad (4.105)$$

and c.c.

Variation of the matrix of  $su(4)$  Cartan forms under the coordinate-dependent conformal supersymmetry transformations is presented in the following form

$$\delta_{\xi} \Omega_{\hat{b}}^{\hat{c}}(d) = \hat{J}_{\hat{b}}^{\hat{c}\mu a} d\xi_{\mu a} - \hat{J}_{\hat{b}}^{\hat{c}\mu} d\bar{\xi}_{\mu}^a + i \left( \Omega_{\hat{b}}^{\hat{d}}(d) (\widehat{W}|_{\xi})_{\hat{d}}^{\hat{c}} - (\widehat{W}|_{\xi})_{\hat{b}}^{\hat{d}} \Omega_{\hat{d}}^{\hat{c}}(d) \right) - d(\widehat{W}|_{\xi})_{\hat{b}}^{\hat{c}}, \quad (4.106)$$

where the current contributions

$$\hat{J}_b^{\hat{c}\mu a} = \frac{\partial}{\partial \xi_{\mu a}} \Omega_{\hat{b}}^{\hat{c}}(\delta_\xi) = \begin{pmatrix} \hat{j}_b^{c\mu a} & \hat{j}_{bc}^{\mu a} \\ \hat{j}_{bc\mu a} & -\hat{j}_c^{b\mu a} \end{pmatrix} \quad (4.107)$$

and

$$\hat{\bar{J}}_b^{\hat{c}\mu}{}_a = -\frac{\partial}{\partial \xi_\mu^a} \Omega_{\hat{b}}^{\hat{c}}(\delta_\xi) = \begin{pmatrix} \hat{j}_b^{c\mu}{}_a & -\hat{j}_{bc}^{\mu}{}_a \\ -\hat{j}_{bc\mu}^{}_a & -\hat{j}_c^{b\mu}{}_a \end{pmatrix} \quad (4.108)$$

are obtained by the  $T$ -transformation of the matrices

$$\begin{aligned} J_b^{\hat{c}\mu a} &= \frac{\partial}{\partial \xi_{\mu a}} ((\widetilde{W}|_\xi)_{\hat{b}}^{\hat{c}} + \Psi_{\hat{b}}^{\hat{c}}(\delta_\xi)) = 2(\delta_b^a \Theta^{\mu\hat{c}} - \delta^{a\hat{c}} \Theta_b^\mu) - 4i\bar{\theta}^{\nu a} (\eta_{\nu\hat{b}} \Theta^{\mu\hat{c}} + \Theta_b^\mu \eta_{\nu}^{\hat{c}}), \\ \bar{J}_b^{\hat{c}\mu}{}_a &= -\frac{\partial}{\partial \xi_\mu^a} ((\widetilde{W}|_\xi)_{\hat{b}}^{\hat{c}} + \Psi_{\hat{b}}^{\hat{c}}(\delta_\xi)) = 2(\delta_a^{\hat{c}} \Theta_b^\mu - \delta_{\hat{b}a} \Theta^{\mu\hat{c}}) + 4i\theta_a^\nu (\eta_{\nu\hat{b}} \Theta^{\mu\hat{c}} + \Theta_b^\mu \eta_{\nu}^{\hat{c}}). \end{aligned} \quad (4.109)$$

So that the current contribution matrices (4.107) and (4.108) acquire the form

$$\hat{J}_b^{\hat{c}\mu a} = (T J^{\mu a} \bar{T})_{\hat{b}}^{\hat{c}} = 2(\mathcal{T}_b^a \hat{\Theta}^{\mu\hat{c}} - \hat{\Theta}_b^\mu \mathcal{T}^{\hat{c}a}) \quad (4.110)$$

and

$$\hat{\bar{J}}_b^{\hat{c}\mu}{}_a = (T \bar{J}_a^\mu \bar{T})_{\hat{b}}^{\hat{c}} = 2(\hat{\Theta}_b^\mu \mathcal{T}_a^{\hat{c}} - \mathcal{T}_{\hat{b}a} \hat{\Theta}^{\mu\hat{c}}). \quad (4.111)$$

Then the variation under conformal supersymmetry of the  $\mathbb{CP}^3$  components of the supervielbein is brought to the form

$$\delta_\xi \Omega_b^{\hat{c}\mu a}(d) = (\hat{j}_b)^{\mu a} d\xi_{\mu a} + (\hat{j}_b)^{\mu}{}_a d\bar{\xi}_\mu^a + i(\hat{w}|_\xi)_c^c \Omega_b^{\hat{c}\mu a}(d) - i(\hat{w}|_\xi)_b^c \Omega_c^{\hat{c}\mu a}(d) \quad (4.112)$$

and c.c. expression.

The variation of Cartan forms associated with the super-Poincare generators can be extracted from the general expression (3.26)

$$\delta_\xi \hat{\omega}_b^\nu(d) = j_b^{\nu\mu a} d\xi_{\mu a} + \bar{j}_{\hat{b}a}^{\nu\mu} d\bar{\xi}_\mu^a + \frac{1}{4} \hat{\omega}_b^\lambda(d) (\hat{l}|_\xi)_\lambda{}^\nu + (\hat{b}|_\xi)^{\nu\lambda} \hat{\chi}_{\lambda\hat{b}}(d) - i(\widehat{W}|_\xi)_{\hat{b}}^{\hat{c}} \hat{\omega}_{\hat{c}}^\nu(d) \quad (4.113)$$

with the current contributions given by

$$j_b^{\nu\mu a} = \frac{\partial \hat{\omega}_b^\nu(\delta_\xi)}{\partial \xi_{\mu a}} = e^{-\varphi} (\mathcal{T}_b^a \hat{x}^{\nu\mu} + 2i\bar{\theta}^{\nu a} \hat{\Theta}_b^\mu) \quad (4.114)$$

and c.c. Similarly the variation of Cartan forms associated with the conformal supersymmetry generators reads

$$\delta_\xi \hat{\chi}_{\nu\hat{b}}(d) = J_{\nu\hat{b}}^{\mu a} d\xi_{\mu a} + \bar{J}_{\nu\hat{b}a}^\mu d\bar{\xi}_\mu^a - \frac{1}{4} (\hat{l}|_\xi)_\nu{}^\lambda \hat{\chi}_{\lambda\hat{b}}(d) + (\hat{b}|_\xi)_{\nu\lambda} \hat{\omega}_b^\lambda(d) - i(\widehat{W}|_\xi)_{\hat{b}}^{\hat{c}} \hat{\chi}_{\nu\hat{c}}(d) \quad (4.115)$$

with the corresponding current contributions

$$J_{\nu\hat{b}}^{\mu a} = \frac{\partial \hat{\chi}_{\nu\hat{b}}(\delta_\xi)}{\partial \xi_{\mu a}} = e^\varphi (\Lambda_{-\nu}^\mu \mathcal{T}_b^a + 2i\bar{\eta}_\nu^a \hat{\Theta}_b^\mu - ie^\varphi (\bar{\eta}\eta) \varepsilon_{\nu\lambda} j_b^{\lambda\mu a}) \quad (4.116)$$

and c.c.

As a result current density associated with the superstring action (2.5) invariance under conformal supersymmetry takes the form

$$\mathcal{J}^{i\mu a}(\tau, \sigma) = \mathcal{J}_{AdS}^{i\mu a} + \mathcal{J}_{CP}^{i\mu a} + \mathcal{J}_{WZ}^{i\mu a} \quad (4.117)$$

and c.c. one. Current contributions (4.105) enter the AdS part of the current density

$$\mathcal{J}_{AdS}^{i\mu a} = -\sqrt{-g}g^{ij} \left( \frac{1}{4}(\hat{\omega}_{jm} + \hat{c}_{jm})j^{m\mu a} + \Delta_j j^{\mu a} \right), \quad (4.118)$$

those entering Eq.(4.112) determine the  $\mathbb{CP}^3$  part of the superconformal current density

$$\mathcal{J}_{CP}^{i\mu a} = \frac{1}{2}\sqrt{-g}g^{ij}(\Omega_{jb}{}^4 (*\hat{j}^b)^{\mu a} - \Omega_{j4}{}^b (*\hat{j}_b)^{\mu a}). \quad (4.119)$$

The form of the WZ term contribution is obtained by substituting Eqs.(4.114) and (4.116)

$$\mathcal{J}_{WZ}^{i\mu a} = \frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{b}}^{\hat{c}}(\hat{\omega}_j^{\nu\hat{b}}\varepsilon_{\nu\lambda}\hat{j}_{\hat{c}}^{\lambda\mu a} + \hat{\chi}_{j\nu}^{\hat{b}}\varepsilon^{\nu\lambda}J_{\lambda\hat{c}}^{\mu a}). \quad (4.120)$$

## 5 Conclusion

The  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset sigma-model [9], [10] describes manifestly classically integrable part of the  $AdS_4 \times \mathbb{CP}^3$  superstring action [12]. By virtue of the isomorphism between the  $osp(4|6)$  superalgebra and  $D = 3 \mathcal{N} = 6$  superconformal algebra it can be presented in the conformal basis for  $osp(4|6)/(so(1,3) \times u(3))$  Cartan forms [21] that are identified with the 'reduced'  $(10|24)$ -dimensional  $D = 10 \mathcal{N} = 2A$  superspace vielbein obtained from the full one [12] by setting to zero 8 fermionic coordinates related to space-time supersymmetries broken by the  $AdS_4 \times \mathbb{CP}^3$  superbackground. The  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset sigma-model action is by construction invariant under the global  $OSp(4|6)$  supergroup transformations and hence is also invariant under  $D = 3 \mathcal{N} = 6$  superconformal symmetry that is the global symmetry of ABJM gauge theory [34]. In this paper we have derived explicit expressions for the corresponding world-sheet current densities associated with each type of the transformations from  $D = 3 \mathcal{N} = 6$  superconformal symmetry. Considering the  $OSp(4|6)/(SO(1,3) \times U(3))$  supercoset element parametrized by  $D = 3 \mathcal{N} = 6$  super-Poincare coordinates supplemented by the  $\mathbb{CP}^3$  coordinates,  $AdS_4$  space bulk coordinate and Grassmann coordinates related to the conformal supersymmetry we have found their full transformations under  $D = 3 \mathcal{N} = 6$  superconformal symmetry. So that passing to the canonical formulation it should be possible to calculate the algebra of associated supercharges.

Among the potential applications of the obtained results one could mention the semiclassical quantization around solutions to the  $OSp(4|6)/(SO(1,3) \times U(3))$  superstring equations of motion [35]. They are also the starting point to examine residual symmetry algebras surviving upon fixing the gauge symmetries of  $OSp(4|6)/(SO(1,3) \times U(3))$  sigma-model action (see, e.g. [36]).

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